Staple!

Ord: __ __

Abstract Algebra MAS4301 4864 **Z-InClass** Prof. JLF King *Touch*: 17Sep2019

Hello. Open brain, closed book/notes. Use $\varphi()$ for the Euler phi-fnc and **C.N** for "cycle notation". Use $k!$ to mean "k factorial". Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

Z7: Show no work.

___ z Professor King sometimes gives freebie questions. *Circle* one: **True** **Right On!** ***Who?***

___ a $\varphi(49000) =$ ______ . Express your answer a product $p_1{}^{e_1} \cdot p_2{}^{e_2} \cdots$ of primes to powers.

___ b A posint N is st. $\mathbf{U}(N)$ is cyclic. Using $\varphi()$, group $\mathbf{U}(N)$ has ______ many generators.

___ c With p prime, the number of order-p elements in $\mathbb{S}_p$ is ______ .

___ d+ Permutation σ has cycle-lengths $35, 7, 5, 5, 4, 4, 4, 6, 6$. $\text{Ord}(\sigma)=$ ______ .
Perm σ is *Circle*: **Even** **Odd**

___ e+ We have $\rho, \pi \in \mathbb{S}_7$ where, in our cycle nota., $\rho := (16)(375)\overline{24}$ and $\pi := (134)(25)\overline{67}$. In std disjoint C.N., then, $\rho \circ \pi =$ ______ .

___ f+ Using ε and products $R^i F^j$, list the elements of $\text{Center}(\mathbb{D}_4)$: ______ .

___ g+ Easily, $\varphi(25)=$ ______ . Consequently, $9^{122} \equiv_{25}$ ______ $\in [0\,..\,25)$. [*Hint:* Fermat, Euler, working mod 25.]

Z8: OSSOP: A (possibly non-Abelian infinite) group G has exactly one nt-proper subgp H. Prove that G is finite cyclic, of order p^2, for some prime p.

Z9: OSSOP: The Turnstile puzzle has N=20 tokens and a spinner of width W=4. Prove rigorously that all patterns are obtainable. You may use, without proof, that A.T.s (adjacent transpositions) in all 20 positions, generate all of $\mathbb{S}_{20}$.

Are all patterns obtainable, with both the spinner-dot UP and spinner-dot DOWN? Prove or disprove!

Bonus: The number of subgroups of group $(\mathbb{Z}_{8100}, +, 0)$ is ______ , Express your answer as a product in a natural way. [*Hint:* $81 = 3^4$.]

End of Z-InClass

Z-Home:	___ ___ ___	430pts
Z7:	___ ___ ___	100pts
Z8:	___ ___	45pts
Z9:	___ ___	65pts
Bonus:	___ ___	15pts
Total:	___ ___ ___	640pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

______ Ord: ___