Staple!

Team: ____

Calc III
MAC2313 3191

Z-Home

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Touch: 16Nov2017

Note. The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up(s) to each essay question.

General instructions are on the CHECKLIST. Page #s without citation refer to our textbook. If you find an unfamiliar term, look in "Notes & Hints" at the end of this P-S. *Each member* of the team must retain a *complete copy* of the team's P-S and write-up, **including diagrams**.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797···.Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Z1: Show no work. Questions (a)–(b) refer to these three vectors/points in $\mathbb{R}^3$:

$$\mathbf{w} := (2, 1, -2)\,, \quad \mathbf{v} := (1, 2, 5)\,, \quad \mathbf{a} := (-1, 3, 3)\,.$$

____ a Compute the norm, $\|\mathbf{w}\| =$ ____ ,and the cross-product, $\mathbf{v}\times\mathbf{w} =$ ␣ , and the projection of $\mathbf{v}$ in the $\mathbf{w}$-direction, $\mathrm{Proj}_{\mathbf{w}}(\mathbf{v}) =$ ␣ .

____ b The distance from the origin to Plane($\mathbf{w}, \mathbf{v}, \mathbf{a}$) equals ____ .

____ c List the six vertices of some reg. octahedron H:

____ .

So cos(dihedral ∡ of H) = ␣ .

____ d The quadratic surface $x^2 + y^2 + 6x + 4y + 2z = z^2 - 11$ is a ____ .

____ e In $\mathbb{R}^4$, let $\mathbb{L}_0$ be the line passing through the origin and the point $Q = (1, 2, 3, 4)$. Let $\mathbb{L}_1$ be the line $t \mapsto (2, -1, 0, 1) + t(5, 0, 1, 2)$. The (orthogonal) distance between lines $\mathbb{L}_0$ and $\mathbb{L}_1$ equals

____ .

____ f Let $\mathbb{L}_3$ be the line $3 - x = \frac{y-6}{2} = z - 3$. Let $\mathbb{L}_2$ be the line passing through the origin and the point $Q = (4, -8, -4)$. Then
$\mathrm{Dist}(\mathbb{L}_2, \mathbb{L}_3) =$ ____ .

g Compute the **arclength** of the curve
$\mathbf{w}(t) := t^2\hat{\boldsymbol{\imath}} + [\cos(t) + t\sin(t)]\hat{\boldsymbol{\jmath}} + [\sin(t) - t\cos(t)]\hat{\mathbf{k}}$
from $t = 0$ to $t = \boldsymbol{\pi}$.
Arclength= ____ .

h For the param.curve $\mathbf{r}(t) := t^2\hat{\boldsymbol{\imath}} + t\hat{\boldsymbol{\jmath}} + t^3\hat{\mathbf{k}}$ compute the displacement vector $\mathrm{CoC}_{\mathbf{r}}(t) - \mathbf{r}(t)$:

____ .

i Let **A** and **R** be the surf-acc and radius of some nice planet. Then the escape and golden-snitch speeds are: $S_{\mathrm{esc}} =$ ␣ , $S_{\mathrm{gold}} =$ ␣ .
The *period* of the snitch is $L_{\mathrm{gold}} =$ ␣ .

Z2: (Let **A** denote the surf-acc of Earth, in in/sec².) The Boston Science Museum has a curvy-funnel (AoS vertical; with diam about 4ft) Pushing a button releases a ball-bearing (BaB) which rolls angled into the funnel; its path looks like a planetary orbit, because the funnel is shaped correctly. Let $g(r)$ denote the depth of the funnel at radius r (each in inches). Design $g()$ st., for each r, if BaB is rolling at the correct speed to stay at the radius=r circle, then the toward-AoS component of force is

$$\mathbf{A}\cdot\frac{U}{r^2}\,, \qquad \text{(Thanks Isaac!)}$$

where $U := 24 \cdot \mathrm{in}^2$.

So $g(r)$= ____ and BaB-speed(r) = ␣ .

(Answers can be ITOf **A** and U; your units *must* be correct!)

Z3: The pleasant planet Pal has surf-acc **A** and radius **R**. Each length $Y \in [0, \mathbf{R})$ determines a straight *tunnel* through Pal from surface-point (X, Y) to surf-pt $(X, -Y)$, where X is the non-negative length st. $X^2 + Y^2 = \mathbf{R}^2$. Coordinatize the tunnel with a vertical "h-axis" from Y downto $-Y$, with coord-zero at the tunnel's middle.

At time zero, drop a capsule into the tunnel; it accelerates till it reaches the middle, then decelerates, arrives at $-Y$, and then reverses the process. Compute the period L_Y of this oscillation.

Let $h_Y(t)$ denote the h-coord of the capsule at time t; so

D1: $\quad h_Y(0\mathrm{sec}) = Y \quad$ and $\quad h'_Y(0\mathrm{sec}) = 0\frac{\mathrm{ft}}{\mathrm{sec}}\,.$

Derive a formula for this interesting parameterization; $h_Y() = $ ␣.

Its maximum speed $S_{\max,Y}$ is ....

What "coincidences" do you notice? —can you explain them?

Notes & Hints. (Be aware that Marston Science Library has mathematical dictionaries.) There is a convenient way to place a cube in 3-space. Since the octahedron is the dual of a cube, there is an easy way to place an octahedron in space. The dihedral angle at an edge of a polyhedron, is the angle between the two *faces* which meet at that edge. (You'll need to think about exactly what this means.)

You'll probably want to cite (and hence will need to look up) Newton's thm on the gravitational field inside a unif.density sphere.

End of Z-Home

Z1:	___ ___ ___	180pts
Z2:	___ ___	80pts
Z3:	___ ___	90pts
Total:	___ ___ ___	350pts

Honor Code: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

Ord: ....

Ord: ....

Ord: ....