Staple!

Team:_____

Calc III
MAC3474 3129

Z-Home

Prof. JLF King
Touch: 31Jul2016

Note. The order of your hand-in should be: PROBLEM SHEET (this side up), NOTATION SHEET (if any), followed by the write-up(s) to each essay question.

General instructions are on the CHECKLIST. Page numbers without citation refer to our textbook. If you find an unfamiliar term, look in the "End Notes and Hints" section at the end of this problem-sheet. Photocopy your write-up before turning it in; each member of the team must retain a *complete copy* of the team's problem-sheet and write-up, including diagrams.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797⋯.Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Z1: Show no work. Questions (a)–(b) refer to these three vectors/points in $\mathbb{R}^3$:

$$\mathbf{w} := (-2, 2, 1)\,, \quad \mathbf{v} := (1, 2, 3)\,, \quad \mathbf{a} := (-1, 3, 3)\,.$$

__ a Compute the norm, $\|\mathbf{w}\| =$ ____ ,and the cross-product, $\mathbf{v} \times \mathbf{w} =$ ____ ,
and the projection of $\mathbf{v}$ in the $\mathbf{w}$-direction, $\text{Proj}_{\mathbf{w}}(\mathbf{v}) =$ ____ .

__ b Compute the distance from the origin to the plane passing through $\mathbf{w}$, $\mathbf{v}$ and $\mathbf{a}$. Dist= ____ .

__ c Let α denote the angle between a hyperdiagonal and an edge of a cube. So $\cos(\alpha) =$ ____ .

__ d Let β denote the *dihedral* angle of a regular tetrahedron. Then $\cos(\beta) =$ ____ .

__ e Let $\mathbb{L}_0$ be the line $\dfrac{x}{2} = \dfrac{y}{-2} = -z$. Let $\mathbb{L}_1$ be the line $x+4 = \frac{y-1}{2} = \frac{1-z}{2}$. Compute the (orthogonal) distance between lines $\mathbb{L}_0$ and $\mathbb{L}_1$.
Distance($\mathbb{L}_0, \mathbb{L}_1$)= ____ .

__ f Let $\mathbb{L}_2$ be the line passing through the origin and the point $Q = (-3, 6, 3)$. Let $\mathbb{L}_3$ be the line
$3 - x = \frac{y-6}{2} = z - 3$.
Distance($\mathbb{L}_2, \mathbb{L}_3$)= ____ .

__ g Compute the **arclength** of the curve $\mathbf{w}(t) := t^2\widehat{\imath} + [\cos(t) + t\sin(t)]\widehat{\jmath} + [\sin(t) - t\cos(t)]\widehat{\mathbf{k}}$ from $t = 0$ to $t = \pi$.
Arclength= ____ .

Z2: You are on a planet whose acceleration due to gravity is $A\frac{\text{feet}}{\text{sec}^2}$ (down!). This planet has a mountain top on which there is a cannon which can fire in every direction. It has a known muzzle speed of $\sigma\frac{\text{feet}}{\text{sec}}$. Compute the "envelope of safety", $y = f(x)$, for this cannon. Carefully graph the envelope, and show some cannonball trajectories which come tangent to it.

Note: σ and A are positive.

ASIDE: The **envelope of safety** is a certain curve $y = f(x)$. Each point *outside* this curve cannot be hit by a cannonball, no matter how the cannon is aimed. Each point *inside* or *on* this curve (that is, every point (x, y) with $y \leqslant f(x)$) can be hit by a cannonball, if the cannon is appropriately aimed. I'm asking you to come up with a specific formula for $f()$.

Here is one approach that will work –perhaps you can find a shorter approach?

Step S1. Let the "θ-cannonball" mean a cannonball fired at angle θ, relative to the vertical.

Fix a non-zero number x. As a function of θ, compute the time –call it t_x– when the θ-cannonball crosses the vertical line through $(x, 0)$. Now compute the *height* where the cannonball crosses the vertical line, and call this height $h(\theta)$.

Step S2. Now find *the* (it turns out to be unique) value of θ which maximizes $h(\theta)$; call this particular value α. Thus α is some particular function of x.

Step S3. Now *argue carefully* that $f(x)$ must in fact equal $h(\alpha)$. Also handle the case $x = 0$.

Step S4. Give *convincing* **qualitative** arguments that your formula for $f()$ is correct. Note that your formula depends on σ and A; let us indicate the dependence by writing the envelope function as $f_{\sigma,A}()$.

From your intuition of the situation, how do you expect $f_{\sigma,A}()$ to vary as you change σ and A? For example, between $f_{3\sigma,A}()$ and $f_{\sigma,A}()$ what relation to you expect? (If you are smart here, you will think carefully about this *before* ever computing f.)

Notes and Hints. (Be aware that Marston Science Library has mathematical dictionaries.) For the cannonball problem, you may find it beneficial to first solve the special case when $\sigma = 1$ and $A = 1$. Then look at the algebra you did and re-work it with letters "σ" and "A".

There is a convenient way to place a regular tetrahedron in 3-space, using four vertices of a cube. The dihedral angle at an edge of a polyhedron, is the angle between the two *faces* which meet at that edge. (You'll need to think about exactly what this means.)

End of Z-Home

Z1:	___ ___ ___	230pts
Z2:	___ ___ ___	110pts
Total:	___ ___ ___	340pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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