Number Theory
MAS 4203

InClass-Z

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Touch: 3Aug2016

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$Z1$:	___ ___ ___	150pts
$Z2$:	___ ___	60pts
Bonus:	___ ___	20pts
Total:	___ ___ ___	210pts

Please PRINT your name and ordinal. Ta:

Ord: ....

$Z1$: Short answer. Show no work.

a Give an example of integers $x, y > 100$, neither prime, so that $x \perp y$.

$x =$ .... and $y =$ ....

b Consider a tuple $\mathbf{b} = \langle b_{M-1}, b_{M-2}, \ldots, b_1, b_0 \rangle$ of coefficients $b_j \in [-1 \,..\, 1]$, such that $b_{M-1} \neq 0$ and

$$\sum_{j=0}^{M-1} b_j \cdot 3^j \quad = \quad 248\,.$$

Then $M =$ .... and $\mathbf{b} = \langle$ .... $\rangle$.

Note. The next questions refer to mapping $f{:}\mathbb{Z}{\to}\mathbb{Z}$ defined by $f(N) := t - 2u$, where we have written N as $N = 10t + u$ with $t \in \mathbb{Z}$ and $u \in [0 \,..\, 9]$.

c Compute please: $f^{\circ 3}(57218) =$ ....

d If $f(K) = 18$, then a possible value for K is ..... .

e Applying f nine times to N, we discover that

$$f^{\circ 9}(N) \quad \equiv_7 \quad 1\,.$$

Letting b be the value in $[-3 \,..\, 3]$ such that $N \equiv_7 b$, we have that $b =$ ....

Note. For the following problem please carefully write up your solution on separate sheets of paper. Show all work —there *is* partial credit.

$Z2$: You may use, without proof, that if $x \perp y$ then there exists integers S and T so that $xS + yT = 1$.

Prove this: If $a \neq 0$ and b, c are integers with $a \perp b$, then

$$a \bullet\!\!\mid bc \quad \Longrightarrow \quad a \bullet\!\!\mid c\,.$$

End of InClass-Z

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....