Sets and Logic
MHF3202 8768

Prac-prereq-

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Hello. *This practice prereq is to help you self-evaluate on material you should review.*

Without reviewing anything, no books, no internet, take 65 minutes to do as many of the problems below that you can. *Then relax for a while...* Then use books/internet to grade yourself.

Short-answer: Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(\text{-}2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = \text{-}2$, yet $[1/g](3) \stackrel{\text{def}}{=} 1/g(3) = 1/8$.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

ℓ1
The **slope** of line $3[y - 5] = 2[x - 2]$ is ....

Point $(\text{-}4, y)$ lies on this line, where $y=$ ....

ℓ2
Line $y = [M \cdot x] + B$ owns points $(4, 3)$ and $(\text{-}2, 5)$.
Hence $M=$ .... and $B=$ ....

ℓ3
Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(2, 1)$. So $M=$ .... and $B=$ ....

q1
The solutions to $3x^2 = 2 - 2x$
are $x =$ ....

q2
The four solutions to $[y - 2] \cdot y \cdot [y + 2] = \text{-}1/y$
are $y =$ ....

[*Hint*: Apply the Quadratic Formula to y^2.]

e1
$\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}} =$ .... $\log_{64}(16)=$ ....

i
Let $y = f(x) := [2 + \sqrt[5]{x}]/3$. Its inverse-function is $f^{-1}(y)=$ ....

id1
Suppose g is a fnc with g' never zero. Let h be the inverse-fnc of g. In terms of h, g, g' and x, write a formula for $h'(x)=$ ....

[*Hint*: The Chain rule. NOTE: h is **NOT** $1/g$.]

id2
Let $g(x) := x^3 + x$. Then $g^{-1}(10)=$ ....

and $[g^{-1}]'(10)=$ ....

de2
On those x where $\sin(x) > 0$, define $B(x) := [\sin(x)]^x$. Its derivative is

$B'(x)=$ ....

[*Hint*: How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

dc1
Below, f and g are differentiable fncs with

$$f(2) = 3, \quad f(3) = 5, \quad f'(2) = 19, \quad f'(3) = 17,$$
$$g(2) = 11, \quad g(3) = 13, \quad g'(2) = \tfrac{1}{2}, \quad g'(3) = 7,$$
$$f(5) = 43, \quad g(5) = 23, \quad f'(5) = 41, \quad g'(5) = 29 \,.$$

Define the composition $C := g \circ f$. Then
$C(2) =$ .... ; $C'(2) =$ ....

Please write each answer as a product of numbers; **do not** multiply out. [*Hint*: The Chain rule.]

sg1
Compute the sum of this geometric series:
$$\sum_{n=3}^{\infty} [\text{-}1]^n \cdot [3/5]^n =$$ ....

sg2
For natural number K, the sum
$$\sum_{n=3}^{3+K} 4^n \text{ equals}$$ ....

sg3
$\sum_{n=1}^{\infty} r^n = \dfrac{5}{8}$. So $r=$ .... or **DNE**.

[*Hint*: The sum starts with n at **one**, not zero.]

sc
The series $\sum_{k=1}^{\infty} \dfrac{[\text{-}1]^k}{\log(k)}$ (circle one): Diverges,

Converges absolutely, Converges conditionally.

pf
Partial-fraction decomposition:
$$\frac{x + 1}{x^2 + x - 2} = \text{.........} + \text{.........} .$$

Short-answer: Sets&Logic stuff:

B
Binomial coefficient $\dbinom{7}{4}=$ .... = ...

C
Relation $\mathbf{R}$ is a binrel on set $\mathbb{Z}_+$, defined by $x\mathbf{R}y$ IFF $x^2 = 5y$.

Assertion "*Relation $\mathbf{R}$ is reflexive*" is T F
Assertion "*Relation $\mathbf{R}$ is antireflexive*" is T F

As
On $\mathbb{Z}$, the statment "operator $+$ is *associative*" means: *For all* .... *in* .... *it is the case that*

....