Staple!

Sets and Logic
MHF3202 8768

Prac-prereq-

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Hello. *This practice prereq is to help you self-evaluate on material you should review.*

Without reviewing anything, no books, no internet, take 65 minutes to do as many of the problems below that you can. *Then relax for a while...* Then use books/internet to grade yourself.

Short-answer: Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(-2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = -2$, yet $[1/g](3) \stackrel{\text{def}}{=} 1/g(3) = 1/8$.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

ℓ1 The **slope** of line $3[y-5] = 2[x-2]$ is ______.
Point $(-4, y)$ lies on this line, where $y=$ ______.

ℓ2 Line $y = [M \cdot x] + B$ owns points $(4,3)$ and $(-2,5)$. Hence $M=$ ______ and $B=$ ______.

ℓ3 Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(2,1)$. So $M=$ ______ and $B=$ ______.

q1 The solutions to $3x^2 = 2 - 2x$
are $x =$ ______.

q2 The four solutions to $[y-2] \cdot y \cdot [y+2] = -1/y$
are $y =$ ______.
[*Hint:* Apply the Quadratic Formula to y^2.]

e1 $\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}} =$ ______. $\log_{64}(16) =$ ______.

i Let $y = f(x) := [2 + \sqrt[5]{x}]/3$. Its inverse-function is $f^{-1}(y)=$ ______.

id1 Suppose g is a fnc with g' never zero. Let h be the inverse-fnc of g. In terms of h, g, g' and x, write a formula for $h'(x)=$ ______.
[*Hint:* The Chain rule. NOTE: h is **NOT** $1/g$.]

id2 Let $g(x) := x^3 + x$. Then $g^{-1}(10)=$ ______
and $[g^{-1}]'(10)=$ ______.

de1 For $x > 0$, let $B(x) := x^x$. Its derivative is
$B'(x)=$ ______.

[*Hint:* How is y^z, for $y > 0$, defined in terms of the exponential fnc?]

a For $x > 0$, let $B(x) := x^{\sin(x)}$. Hence its derivative is $B'(x) = B(x) \cdot M(x)$, where $M(x)$ equals ______.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

de2 On those x where $\sin(x) > 0$, define $B(x) := [\sin(x)]^x$. Its derivative is
$B'(x)=$ ______.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

dc1 Below, f and g are differentiable fncs with

$$\begin{array}{llll} f(2) = 3, & f(3) = 5, & f'(2) = 19, & f'(3) = 17, \\ g(2) = 11, & g(3) = 13, & g'(2) = \frac{1}{2}, & g'(3) = 7, \\ f(5) = 43, & g(5) = 23, & f'(5) = 41, & g'(5) = 29\,. \end{array}$$

Define the composition $C := g \circ f$. Then
$C(2) =$ ______ ; $C'(2) =$ ______.

Please write each answer as a product of numbers; **do not** multiply out. [*Hint:* The Chain rule.]

sg1 Compute the sum of this geometric series: $\sum_{n=3}^{\infty} [-1]^n \cdot [3/5]^n =$ ______.

sg2 For natural number K, the sum
$\sum_{n=3}^{3+K} 4^n$ equals ______.

sg3 $\sum_{n=1}^{\infty} r^n = \frac{5}{8}$. So $r=$ ______ or **DNE**.
[*Hint:* The sum starts with n at **one**, not zero.]

di1 With $F(t) := \int_{\sin(t^3)}^{\exp(5t)} \cos(\sin(x))\, dx$, then $F'(t)$ equals
______.
Simplified, $F'(0) =$ ______.
[*Hint:* Chain rule and Fund. Thm of Calculus.]

sc The series $\sum_{k=1}^{\infty} \frac{[-1]^k}{\log(k)}$ (circle one): Diverges, Converges absolutely, Converges conditionally.

pf Partial-fraction decomposition:

$$\frac{x+1}{x^2+x-2} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}.$$

As On $\mathbb{Z}$, the statment "operator $+$ is *associative*" means: *For all* ______ *in* ______ *it is the case that*

______________________________.