Abstract Algebra
MAS4301 4864

Y-InClass

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Hello.   Open brain, closed book/notes. Use $\varphi()$ for the Euler phi-fnc and **C.N** for "cycle notation". Answers can be given ITOf factorial, multinomial coeffs, and powers.

Y6:   Short answer: <u>Show no work</u>. Write **DNE** <u>if</u> the object does not exist or the operation cannot be performed. NB: **DNE** $\neq$ {} $\neq$ 0 $\neq$ *Empty-word*.

___ ___ **z**   In a commutative group, STOP equals ⬚ *Circle* ⬚ one:
TOPS   SPOT   POTS   OPTS   ꟼOTꙄ

___ ___ **a**   In 4×4 TTT, with 1st move in the corner, the number of *Really Different* 2nd moves is _____ .

___ ___ **b**   In $\mathbb{S}_{10}$, the max-EltOrd is _____ . Each elt with this order has cycle-lengths _____ .

___ ___ **c⁺**   In $\mathbb{S}_6$, the two CLSs (Cycle-Length Structures) that yield order-6 elements are:
CLS1= _____   and CLS2= _____ .
The **number of elements** with each CLS is
#Elts1= _____   and #Elts2= _____ .

___ ___ **d⁺**   Create an elt $g \in \mathbb{A}_{11}$ with $g^{-1}\alpha g = \beta$, where

$$\alpha := (1\ 2\ 3)(4\ 5\ 6\ 7)(8\ 9\ 10\ 11) \ ;$$
$$\beta := (1\ 3\ 2)(4\ 5\ 7\ 6)(8\ 9\ 11\ 10) \ .$$

In <u>std</u>-C.N, $g=$ _____ .

___ ___ **e**   Easily, $\varphi(125)=$ _____ .   Consequently,

$123^{2005} \equiv_{125}$ _____ $\in [-62 .. 62].$

[*Hint:* Fermat, Euler, working mod 125.]

___ ___ **f**   In $\mathbb{Z}_7 = [-3 .. 3]$, give an r.i (reasonable interpretation) to $\frac{3}{5}=$ _____ , $\sqrt{-2}=$ _____ .
All values that <u>are</u> a r.i of $\sqrt[3]{1}$: _____ .

Y7:   Let $\Gamma := [\![G, G]\!]$ denote the the commutator subgp of G.

i   Prove that Γ is *strongly normal* (characteristic) in G. [*Hint:* I.e, that each $\alpha \in \text{Aut}(G)$ preserves Γ.]

ii   Prove that G/Γ is abelian.

iii   Consider a subgp $N \lhd G$, with $\frac{G}{N}$ abelian. Prove that $N \supset \Gamma$.

iv   Imagine a subgp H with $\Gamma \subset H \subset G$. Prove that $H \lhd G$. [*Hint:* Does ISTShow, given $g \in G$ and $h \in H$, that $ghg^{-1}H \subset H$?]

Y8:   <u>**State**</u> the Orbit-Stabilizer Theorem, where a gp G acts on a set Ω, both finite. <u>**Prove**</u> the O-S Thm. [*Hint:* PC for defining what it means for G to "**act on**" Ω; what an **orbit** (of what?) is; what a **stabilizer** (of what?) is.]

Y9:   State Dirichlet's Thm.

Bonus: Prove that $\text{Inn}(G) \lhd \text{Aut}(G)$. [*Hint:* Just chase the definitions.]

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│ No groups were harmed in the making of this exam. │
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Y-Home:	__ __ __	495pts
Y6:	__ __	90pts
Y7:	__ __	110pts
Y8:	__ __	85pts
Y9:	__ __	15pts
Bonus:	__ __	15pts
Total:	__ __ __	795pts

Ord: _____