Staple!

Team: ____

Calc III
MAC2313 3191

Y-Home

Prof. JLF King
Touch: 6May2016

Note. The order of your hand-in should be: PROBLEM SHEET (this side up), NOTATION SHEET (if any), followed by the write-up(s) to each essay question.

General instructions are on the CHECKLIST. Page numbers without citation refer to our textbook. If you find an unfamiliar term, look in the "End Notes and Hints" section at the end of this problem-sheet. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797···. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Y1: Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

____ **a** A cannon fires a cannonball up at an angle of $\frac{\pi}{6}$ (relative to the horizontal) at 100 feet per second. The cannon sits at the edge of a 500 foot cliff and fires the cannonball over the plain below. How far (horizontally) from the **base** of the cliff will the cannonball land, when it hits the plain?
Distance≈ .... feet.

____ **b** Let $\mathbf{w} := \frac{1}{\sqrt{2}}[\hat{\imath} - \hat{\jmath}]$. At point $Q := (3,2)$, find this directional derivative of $f(x,y) := 7x^2 - y^2$:
$[D_\mathbf{w} f](Q) =$ ....

____ **c** Write an eqn for the plane which is tangent to $x^2 - y^2 + z^2 = 0$ at the point $(3,5,4)$. Write the plane in the form $\alpha[x-x_0] + \beta[y-y_0] + \gamma[z-z_0] = 0$.
Plane: .... $= 0$.

____ **d** Let $x := u + 2^{uv}$ and $y := uv$. Compute the Jacobian. $\dfrac{\partial(x,y)}{\partial(u,y)}\left(\sqrt{3}, \sqrt{12}\right) =$ ....

____ **e*** Fix $L > 0$. The **surface area** of the surface $z = xy$ lying *inside* the cylinder $x^2 + y^2 = L^2$ is ....

____ **f*** An object in the first octant occupies a cube of volume 8 with three of its faces on the coordinate planes. The mass-density at each point in the cube is equal to the square of the point's distance from the origin, so the **total mass** of the object is ....

g* In the plane, the four points $(0,0)$, $(0,5)$, $(4,2)$, $(4,-3)$ are the corners of a rhombus. Let B denote the region enclosed by this rhombus, including its boundary. For the function $\varphi(x,y) := xy + 5 - 3y - 2x$ on the region B, find (the location of): a **global** max; a **global** min; and a saddle point.
$P_{\text{max}} =$ .... $P_{\text{min}} =$ .... $P_{\text{saddle}} =$ ....

Y2: Essay Questions: In $\mathbb{R}^3$, let $\mathbf{Y}_1$ and $\mathbf{Y}_2$ be (infinite) solid cylinders, each of radius 1, whose axes intersect; let $\theta \in (0, \frac{\pi}{2}]$ denote the angle between the axes. Let

$$\mathbf{S} \;=\; \mathbf{S}(\theta) \;:=\; \mathbf{Y}_1 \cap \mathbf{Y}_2$$

denote the resulting solid of intersection.

α For several values of θ, draw LARGE carefully labeled and shaded pictures of $\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{S}(\theta)$. Certain cross-sections of the cylinders are useful; draw them too. For the surface of $\mathbf{S}$, inscribe and circumscribe some familiar polyhedra, and use these to give *lower/upper bounds* on the volume of $\mathbf{S}$.

β $\text{Vol}(\mathbf{S}(\theta)) =$ ....
Explain carefully all the ideas on how you computed this; I'll bet that you'll use lots of pictures.

γ Compute the *surface area* of $\mathbf{S}$.

$\text{SurAr}(\mathbf{S}(\theta)) =$ ....
Explain all the ideas on how you computed this. [*Hint:* There is a way to do this with very very little computation, because the surface is made up of pieces of a cylinder.] [*Hint:* Can you get interesting lower/upper bounds on this surface-area?]

Y3: Call a largest dimension of a rectangular box its ***length***. (If two or all three of the dimensions of the box are equal (and longest), pick one and call it the "length".) The ***girth*** of the box is the perimeter of the rectangular cross-section perpendicular to the "length" direction.

Parcel Post (PP) has changed its shipping rules. PP has fixed a positive real number K, and now insists that any rectangular package shipped satisfy that its length plus K times its girth be less or equal 160inches. (I.e Len+[K·girth].)

a Let $v(K)$ denote the volume of the package of largest volume that Parcel Post will ship. What do you expect $\lim_{K\to\infty} v(K)$ to equal? Why? What do you expect $\lim_{K\to 0^+} v(K)$ to equal? Why?

b Carefully derive a formula for $v(K)$, explaining your method in detail. Good pictures **will** be useful.

c As a function of K, tell me the dimensions (length, width, height) of the/a box of volume $v(K)$ that PP will ship. In other words, tell me the dimensions of the box that allowed you to solve part (b).

d There is a "critical value" of K above which the formula for $v(K)$ behaves one way (mathematically), and below which it behaves another way. What is this critical value of K? ***Why*** does this phenomenon happen? Discuss in detail. Make a *careful* ***graph*** of $K \mapsto v(K)$ for $0 \leqslant K < \infty$.

***Bonus*:** Using correct English, invent a *funny*, **original**, clever (non-vulgar) joke or story. A mathematics joke is acceptable eg.

$$\lim_{\text{GPA}\to 0} \textit{Engineering student} \;=\; \textit{Business major}\,.$$

(Of course, the Business school has a different version of this joke...)

Notes and Hints.

(Be aware that Marston Science Library has mathematical dictionaries.)

In parts of Y1, polar coordinates are useful.

The gradient is a convenient way to find a vector which is orthogonal to a level curve or level surface.

The path of a cannonball can be computed by integrating a constant acceleration field (that of the Earth, near the surface) twice.

End of Y-Home

Y1:	___ ___ ___	270pts
Y2:	___ ___	85pts
Y3:	___ ___ ___	105pts
Bonus:	___ ___	10pts
Total:	___ ___ ___	460pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....

Filename: `Classwork/3Calculus/Calc2002t/y-home.3Calc32002t.latex`
As of: *Monday 31Aug2015.* Typeset: *6May2016* at *13:25.*