Staple!

Team:_____

Calc III
MAC2313 3191

Y-Home

Prof. JLF King
Touch: 6May2016

Note. Carefully adhere to the **Checklist**, and to the generic instructions.

Y1: Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ a+ A cannon fires a cannonball up at an angle of $\frac{\pi}{8}$ (relative to the horizontal) at 100 feet per second. The cannon sits at the edge of a 670 foot cliff and fires the cannonball over the plain below. How far (horizontally) from the **base** of the cliff will the cannonball land, when it hits the plain?

Distance$\approx$ _____ feet.

___ b Let $\mathbf{w} := \frac{1}{\sqrt{2}}[\hat{\imath} - \hat{\jmath}]$. At point $Q := (3, 2)$, find this directional derivative of $f(x, y) := 7x^2 - y^2$:

$[D_{\mathbf{w}} f](Q)=$ _____.

___ c Write an equation for the plane which is tangent to $2x^2 + z^2 = y^2 + 9$ at the point $P := (3, 5, 4)$. Write the plane in the form $\alpha[x-x_0]+\beta[y-y_0]+\gamma[z-z_0] = 0$.

Plane: _____ $= 0$.

___ d Compute the following iterated integral.

$$\int_0^{\pi/8} \int_0^{\cos(\theta)} r^4 \sin(\theta)\, dr\, d\theta =$$ _____.

___ è Fix $L > 0$. The **surface area** of the surface $z = xy$ lying *inside* the cylinder $x^2 + y^2 = L^2$ is _____.

___ f An object in the positive octant occupies a cube of volume 8 with three of its faces on the coordinate planes. The mass-density at each point in the cube is equal to the square of the point's distance from the origin, so the **total mass** of the object is _____.

___ g In the plane, the four points $(0, 0)$, $(0, 5)$, $(4, 2)$, $(4, -3)$ are the corners of a rhombus. Let B denote the region enclosed by this rhombus, including its boundary. For the function $\varphi(x, y) := 3y + 2x + 8 - xy$ on the region B, find (the location of): a **global** max; a **global** min; and a saddle point.

$P_{\max}=$ _____ $P_{\min}=$ _____ $P_{\text{saddle}}=$ _____

Y2: Our astronomy community has informed us that a comet, in parabolic orbit about the Sun, is going to arrive closest to Earth in 2 years, plus or minus a few days. Its path, in a std coordinate system, is $2y = x^2$. In this coord system, the Earth's location then will be $\mathbf{E} = (A, B)$. I want you to explore the set of pts on the parabola which are closest to $\mathbf{E}$ (in general, local extrema), as a fnc of $\mathbf{E}$.

Draw careful labeled pictures of the parabola, locating the Sun specifically. Positioning $\mathbf{E}$, how many EXA (extrema) do you see geometrically? How do the number and positions vary AAFOf $\mathbf{E}$? Categorize your EXA as local min/max. Draw some osculating circles of the parabola, and relate RoC to whether an extremum is a local min/max.

As you move $\mathbf{E}$ along AoS from $-\infty$ to $+\infty$, how do EXA vary? (Show me all this visually, with a series of pictures.)

When $\mathbf{E} := (24, 15)$ then the set of extrema is _____.

AAFOf (A, B), describe the general soln to the set of EXA. This will involve finding zeros of a polynomial. Look up the formula of Cardano (also Cardan) and use it to describe a general soln.

Do something extra; something **good**.

***Bonus*:** Using correct English, invent a *funny*, **original**, clever (non-vulgar) joke or story. A mathematics joke is acceptable eg.

$$\lim_{\text{GPA}\searrow 0} \textit{Engineering student} \;=\; \textit{Business major}\,.$$

(Of course, the Business school has a different version of this joke...)

Notes and Hints. (Be aware that Marston Science Library has mathematical dictionaries.)

In parts of Y1, polar coordinates are useful.

The gradient is a convenient way to find a vector which is orthogonal to a level curve or level surface.

The path of a cannonball can be computed by integrating a constant acceleration field (that of the Earth, near the surface) twice.

End of Y-Home

Y1:	___ ___ ___	180pts
Y2:	___ ___	85pts
Bonus:	___ ___	10pts
Total:	___ ___ ___	265pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord:

Ord:

Ord: