



Staple!

Calc III
MAC3474 3129

Y-Home

Prof. JLF King
Touch: 31Jul2016**Note.** The order of your hand-in should be: PROBLEM SHEET (this side up), NOTATION SHEET (if any), followed by the write-up(s) to each essay question.

General instructions are on the CHECKLIST. Page numbers without citation refer to our textbook. If you find an unfamiliar term, look in the “End Notes and Hints” section at the end of this problem-sheet. Do **not** approx.: If your result is “ $\sin(\sqrt{\pi})$ ” then write that rather than $.9797\dots$. Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g. write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$. Write expressions unambiguously e.g., “ $1/a+b$ ” should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Y1: Show no work.

a,b

A cannon fires a cannonball up at an angle of $\frac{\pi}{6}$ (relative to the horizontal) at 100 feet per second. The cannon sits at the edge of a 500 foot cliff and fires the cannonball over the plain below. How far (horizontally) from the **base** of the cliff will the cannonball land, when it hits the plain?

Distance \approx _____ feet.

c

Let $\mathbf{w} := \frac{1}{\sqrt{2}}[\hat{\mathbf{i}} - \hat{\mathbf{j}}]$. At point $Q := (3, 2)$, find this directional derivative of $f(x, y) := 7x^2 - y^2$:

 $[D_{\mathbf{w}}f](Q) =$ _____.

d

Write an equation for the plane which is tangent to $x^2 - y^2 + z^2 = 0$ at the point $(3, 5, 4)$. Write the plane in the form $\alpha[x - x_0] + \beta[y - y_0] + \gamma[z - z_0] = 0$.

Plane: _____ $= 0$.

e

Let $x := u + 2^{uv}$ and $y := uv$. Compute the Jacobian. $\frac{\partial(x, y)}{\partial(u, v)}(\sqrt{3}, \sqrt{12}) =$ _____.

f,g

Compute the **surface area** of the surface $z = xy$ lying **inside** the cylinder $x^2 + y^2 = 9$.

Surface-Area = _____.

h,i

An object in the first octant occupies a cube of volume 8 with three of its faces on the coordinate planes. If the mass density at each point in the cube is equal to the square of the point’s distance from the origin, find the total mass of the object.

Total-mass = _____.

j,k

In the plane, the four points $(0, 0)$, $(0, 5)$, $(4, 2)$, $(4, -3)$ are the corners of a rhombus. Let B denote the region enclosed by this rhombus, including its boundary. For the function $f(x, y) := xy + 5 - 3y - 2x$ on the region B , find (the location of): a **global** max; a **global** min; and a **saddle** point. In the case that one or more of these does not exist (on B), write “DNE” in the corresponding blank.

 $P_{\max} =$ _____ $P_{\min} =$ _____ $P_{\text{saddle}} =$ _____

Y2: Call a largest dimension of a rectangular box its **length**. (If two or all three of the dimensions of the box are equal, and longest, pick one and call it the “length”.) The **girth** of the box is the perimeter of the rectangular cross-section perpendicular to the “length” direction.

Parcel Post (PP) has changed its shipping rules. PP has fixed a positive real number K , and now insists that any rectangular package shipped satisfy that its length plus K times its girth be less or equal 160 inches.

a

Let $v(K)$ denote the volume of the package of largest volume that Parcel Post will ship. What do you expect $\lim_{K \rightarrow \infty} v(K)$ to equal? Why?

b

Carefully derive a formula for $v(K)$, explaining your method in detail. Good pictures **will** be useful.

c

As a function of K , tell me the dimensions (length, width, height) of the/a box of volume $v(K)$ that PP will ship. In other words, tell me the dimensions of the box that allowed you to solve part (b).

d

There is a “critical value” of K above which the formula for $v(K)$ behaves one way (mathematically), and below which it behaves another way. What is this critical value of K ? **Why** does this phenomenon happen? Discuss. Make a *careful graph* of $K \mapsto v(K)$ for $0 \leq K < \infty$.

Bonus: Using correct English, type up a good, funny (clean, non-offensive) joke or story. A mathematics joke is acceptable eg.

$$\lim_{\text{GPA} \rightarrow 0} \text{Engineering student} = \text{Business major}.$$

(Of course, the Business school has a different version of this joke...)

Team: _____

Notes and Hints. (Be aware that Marston Science Library has mathematical dictionaries.)

In parts of Y1, polar coordinates are useful.

The gradient is a convenient way to find a vector which is orthogonal to a level curve or level surface.

The path of a cannonball can be computed by integrating a constant acceleration field (that of the Earth, near the surface) twice.

[End of Y-Home](#)

Y1: _____ 330pts

Y2: _____ 110pts

Bonus: _____ 10pts

Total: _____ 440pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord:

Ord:

Ord:
