

Please. Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g, write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$. Do **not** approx.: If your result is “ $\sin(\sqrt{\pi})$ ” then write that rather than $.9797\ldots$. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Y3: Show no work.

Z Prof. King believes that writing in complete sentences aids in communicating technical ideas. **Circle** one: **True** **What's a sentence?** **DNE**

a Let $P_0 := (2\pi, 1, 2)$. Compute the gradient of $h(x, y, z) := xy + z \cdot \sin(x)$.

$$[\nabla h](P_0) = \underline{\quad}$$

Give an example of a *polynomial* f which has a saddle-point at $R := (-3, 5)$.

$$f(x, y) := \underline{\quad}$$

b In \mathbb{R}^3 , let S be the surface

$$6x - 3x^2 + 3 = 2y^2 + z^2.$$

Vector $\mathbf{v} = \underline{\quad}$ is $\neq \mathbf{0}$ is $\perp S$ at $Q := (0, 1, 1)$.

In form $A[x - x_0] + B[y - y_0] + C[z - z_0] = 0$, write an *equation* for the tangent plane to S at Q .

Eqn:

Have arranged that A, B, C are **integers** with no common factor; also, that $A \geq 0$.

c Let $f(x, y) := x - 14y$. *Subject to the constraint that* $y^2 = x$, compute the location (x_0, y_0) of a **global maximum** of f , and compute the location (x_1, y_1) of a **global minimum** of f .

$$\text{Max} = (\underline{\quad}, \underline{\quad}); \text{ Min} = (\underline{\quad}, \underline{\quad}).$$

d At the point $P := (1, 0)$, the *curvature* of curve $y = \ln(x)$ is $\underline{\quad}$.

e Glued to a massless plate is a 10 lb weight at the origin, a 15 lb weight at the point $(3, -1)$, and 5 lb at point $(\underline{\quad}, \underline{\quad})$, thus putting the center-of-mass of the weighted-plate at $(2, 1)$.

Y4: A cylindrical soup-can has radius r and height y . Let $A(r, y) =$

denote the *surface area* of the can. Note that the volume of the can is $\pi \cdot r^2 \cdot y$.

Subject to the volume being held constant (say, 10 cubic inches), what is the *ratio* of r/y that **minimizes** surface area (i.e, minimizes the cost of the can)? Use Lagrange multipliers.

Let $g(r, y) :=$ denote the specifier fnc that you choose. Compute the three equations (one constraint eqn. followed by two Lagrange eqns.). Use β for your L.Multiplier.

$$C_g : \underline{\quad} = \underline{\quad}.$$

$$L_r : \underline{\quad} = \underline{\quad}.$$

$$L_y : \underline{\quad} = \underline{\quad}.$$

Solve the system to compute $\frac{r}{y} = \underline{\quad}$.

End of Y-class

Y-home: _____ 265pts

Y3: _____ 120pts

Y4: _____ 40pts

Total: _____ 425pts

HONOR CODE: *I have neither requested nor received help on this exam other than from my professor (or his colleague).*
Name/Signature/Ord