



Sets and Logic
MHF3202 8768

Class-Y

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Touch: 17Nov2018

Y1: Short answer. Show no work.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Repeating decimal $2.3\overline{84}$ equals $\frac{n}{d}$, where posints $n \perp d$ are $n=$ _____ and $d=$ _____.

b Let \mathcal{P}_∞ denote the family of all *infinite* subsets of \mathbb{N} . Define relation \bowtie on \mathcal{P}_∞ by: $A \bowtie B$ IFF $A \cap B$ is infinite. Stmt “This \bowtie is an equivalence-relation” is: $\quad T \quad F$

c An explicit bijection $G: [2.. \infty) \hookrightarrow \mathbb{Z}$ is this:
When n is *even*, then $G(n) :=$ _____.
When n is *odd*, then $G(n) :=$ _____.

d 16 and 6 and 13
The map $H(k, n) := 2^k \cdot [1 + 2n]$ is a bijection from $\mathbb{N} \times \mathbb{N} \hookrightarrow \mathbb{Z}_+$. And $H^{-1}(208) = ($ _____, _____).

e A bijection $f: [7, 8] \hookrightarrow (0, 1)$ is: $f(7) :=$ _____;
 $f($ _____) $:=$ _____, for *each* $k \in$ _____;
and $f(x) :=$ _____, for *each* $x \in (7, 8] \setminus C$,
where $C :=$ _____.

f Sequence $\vec{L} := (L_n)_{n=0}^\infty$ is defined by $L_0 := 5$, $L_1 := 4$, and $(\forall n \in \mathbb{N}: L_{n+2} = 2L_{n+1} + 15L_n)$. This implies $(\forall k \in \mathbb{N}: L_k = [P \cdot \alpha^k + Q \cdot \beta^k])$, for real numbers $\alpha =$ _____ $< \beta =$ _____.

OYOP: In grammatical English ***sentences***, write your essay on every ***third*** line (usually), so that I can easily write between the lines. Please put your ordinal also on the back of the last page, **large**, right-side-up.

Y2: **α** The powerset $\mathcal{P}(\Omega)$ of set Ω , is... .

β Give a complete proof that there is **no** surjection $h: \Omega \twoheadrightarrow \mathcal{P}(\Omega)$. In particular, given a map $h: \Omega \rightarrow \mathcal{P}(\Omega)$, explicitly construct a set $S_h \subset \Omega$ which is *guaranteed* to **not** be in the range of h .

γ When $\Omega := \{M, L, C\}$, the three Stooges, consider this map

$$\begin{aligned} g(M) &:= \{M, L, C\}; \\ g(L) &:= \{M, C\}; \\ g(C) &:= \{\}. \end{aligned}$$

Your $S_g = \{$ _____ $\}$.

End of Class-Y

Y1: _____ 140pts

Y2: _____ 55pts

Not triple-spaced: _____ -15pts

Ouch!, scratch work
handed-in : _____ -5pts

Total: _____ 195pts

Please PRINT your *name* and *ordinal*. Ta:

Ord: _____

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature: _____