



Staple!

Team: _____

Abstract Algebra
MAS4301 4864

X-Home

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Touch: 6May2016

Hello. Essays violate the CHECKLIST at *Grade Peril!* Expression “**NSIShow**” stands for “Next semester I’ll show”. Use **SoTS** for “Sum of Two Squares”, e.g, $5 = 1 + 4$ is a SoTS, but 3 is not a SoTS.

Exam is due by **10AM, Friday, 09Dec2005**.

X1: Show no work.

a A monic \mathbb{Q} -poly with $\sqrt[3]{3} + \sqrt[2]{5}$ as a root is _____.

b Write each prime p below as a SoTS, and find $i \in [1..p)$ such that $i^2 \equiv_p -1$.

p	$S_1 + S_2$	i
5	$1 + 4$	2
13		
17		
29		

Next semester I’ll show that oddprime p is SoTS *IFF* -1 is a quadratic residue mod p *IFF* p is of the form $1 + 4n$.

Write $65 = \underline{\quad} + \underline{\quad}$ as a SoTS in two ways, $65 = \underline{\quad} + \underline{\quad}$.

(NSIShow that the number of ways (*caveat*) is 2^{K-1} , where K is the number of 4pos primes in its factorization.)

c Suppose $f(t) := t^2 - St + P = [t - \alpha][t - \beta]$ is a \mathbb{Q} -poly irreducible over \mathbb{Q} ; so $\alpha, \beta \in \mathbb{C} \setminus \mathbb{Q}$. Given rationals x, y NBZero, write $\frac{1}{x+y\alpha}$ as $X + Y\alpha$, where $X = \underline{\quad}, Y = \underline{\quad}$, are explicit rational-expressions in x, y, S, P .

d As polys in $\mathbb{Z}_7[x]$, let

$$f(x) := 2x^4 - 5x^3 - 5x^2 - 16x - 4; \\ g(x) := 6x^3 - 23x^2 + x + 2.$$

Write t.fol polys, using coeffs in $[-3..3]$. Let D be $\text{Ged}(f, g)$. Then $D(x) = \underline{\quad}$.

Compute polys $S(x) = \underline{\quad}, T(x) = \underline{\quad}$, with $Sf + Tg = \bar{D}$.

X2: A gp G acts on a set Ω , both finite. Let $\text{Fix}(g) := \{\omega \in \Omega \mid g\omega = \omega\}$.

i Given a bijective proof that

$$1: \quad \sum_{\omega \in \Omega} |\text{Stab}(\omega)| = \sum_{g \in G} |\text{Fix}(g)|.$$

ii

Use the Orb-Stab Thm and prove that

$$2: \quad \#\text{Orbits} = \frac{1}{|G|} \cdot \sum_{g \in G} |\text{Fix}(g)|.$$

iii

Color the 6-faces of a cube red, white and blue. Use (2) to count, up to (orientation preserving) rotation, the number of distinct 3-colorings. Write $c(N)$, the $\#[N\text{-colorings}]$, as a **polynomial** in N . [Hint: $c(0) = 0$ and $c(1) = 1$]

X3: A **PB** (pegboard) is $\mathbf{B}_{\vec{L}} := [0..L_1] \times \dots \times [0..L_D]$ or, more gen., a finite connected subset $\mathbf{B} \subset \mathbb{Z}^{\times D}$. Some cells are occupied by pegs. A jump goes from $\bullet\bullet\circ$ to $\circ\circ\bullet$, and can be applied in all of the $2D$ directions. A **psn** (position) is a map $\Lambda: \mathbf{B} \rightarrow \{\bullet, \circ\}$; write $\Lambda \rightsquigarrow \Lambda'$ if one can jump from Λ to $\text{psn } \Lambda'$. For a cell $c \in \mathbf{B}$, let c^\bullet mean that c is occupied and all other cells are empty. For cells b, c , write $b \rightsquigarrow c$ if $b^\circ \rightsquigarrow c^\bullet$. Say that b is **blocked** (w.r.t \mathbf{B}) if

there is no $c \in \mathbf{B}$ with $b \rightsquigarrow c$. **a** (Use K4 for the Klein-4 argument.) On $\mathbb{Z}^{\times D}$, what are the K4-equiv-classes, and how many are there? What are the K4-blocked positions in $\mathbf{B}_{\vec{L}}$? In ${}^3+{}^7$? In ${}^W+{}^H$?

b

Is \rightsquigarrow symmetric? Is \rightsquigarrow transitive? DTATTQuestions depend on the board?

c

On $\mathbf{B} := 4 \times 4$, describe all pairs (b, c) st. $b \stackrel{K4}{\rightsquigarrow} c$. Here, is $\stackrel{K4}{\rightsquigarrow}$ the same as \rightsquigarrow ? In $\mathbb{Z}^{\times D}$, two cells are **checker-equiv** if, in principle, checker could jump from one to the other. How many checker-equiv-classes are there? How can you

combine checker-equiv and K4 to find $b \rightarrow c$? **d** Generalize everything. E.g, a **coal**(escence) moves $\bullet\bullet\bullet$ to $\circ\circ\circ$. The K4 jump-invariant is also coal-invariant! On 4×4 , what is $\stackrel{\text{coal}}{\rightsquigarrow}$

Triangle boards? Center-of-mass arguments?

X4: Pick a non-trivial problem from chapters 16–22, and solve it elegantly.

X1: _____ 120pts

X2: _____ 155pts

X3: _____ 165pts

X4: _____ 155pts

Total: _____ 595pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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Filename: Classwork/Algebra/Alg2005t/x-home.Alg2005t.
latex
As of: Monday 31Aug2015. Typeset: 6May2016 at 13:54.