

Abstract Algebra
MAS4301 4864

X-Home

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Hello. Essays violate the CHECKLIST at *Grade Peril!* Expression “**NSIShow**” stands for “Next semester I’ll show”. Use **SoTS** for “Sum of Two Squares”, e.g, $5 = 1 + 4$ is a SoTS, but 3 is not a SoTS.

Exam is due by **10AM, Friday, 09Dec2005.**

X1: Show no work.

a A monic \mathbb{Q} -poly with $\sqrt[3]{3} + \sqrt[3]{5}$ as a root is

b Write each prime p below as a SoTS, and find $i \in [1..p)$ such that $i^2 \equiv_p -1$.

p	$S_1 + S_2$	i
5	$1 + 4$	2
13		
17		
29		

Next semester I’ll show that oddprime p is SoTS *IFF* -1 is a quadratic residue mod p *IFF* p is of the form $1 + 4n$.

Write $65 = \dots + \dots$ as a SoTS in two ways,
 $65 = \dots + \dots$
 (NSIShow that the number of ways (*caveat*) is 2^{K-1} , where K is the number of 4pos primes in its factorization.)

c Suppose $f(t) := t^2 - St + P = [t - \alpha][t - \beta]$ is a \mathbb{Q} -poly irreducible over \mathbb{Q} ; so $\alpha, \beta \in \mathbb{C} \setminus \mathbb{Q}$. Given rationals x, y NBZero, write $\frac{1}{x+y\alpha}$ as $X + Y\alpha$, where $X = \dots$, $Y = \dots$, are explicit rational-expressions in x, y, S, P .

d As polys in $\mathbb{Z}_7[x]$, let

$$f(x) := 2x^4 - 5x^3 - 5x^2 - 16x - 4;$$

$$g(x) := 6x^3 - 23x^2 + x + 2.$$

Write t.fol polys, using coeffs in $[-3..3]$. Let D be $\text{Gcd}(f, g)$. Then $D(x) = \dots$.

Compute polys $S(x) = \dots$,
 $T(x) = \dots$ with $Sf + Tg = D$.

X2: A gp G acts on a set Ω , both finite. Let $\text{Fix}(g) := \{\omega \in \Omega \mid g\omega = \omega\}$.

i Given a bijective proof that

$$1: \sum_{\omega \in \Omega} |\text{Stab}(\omega)| = \sum_{g \in G} |\text{Fix}(g)|.$$

ii Use the Orb-Stab Thm and prove that

$$2: \quad \# \text{Orbits} = \frac{1}{|G|} \cdot \sum_{g \in G} |\text{Fix}(g)|.$$

iii Color the 6-faces of a cube red, white and blue. Use (2) to count, up to (orientation preserving) rotation, the number of distinct 3-colorings. Write $c(N)$, the $\#[N\text{-colorings}]$, as a **polynomial** in N . [*Hint*: $c(0) = 0$ and $c(1) = 1$]

X3: A **PB** (pegboard) is $\mathbf{B}_{\vec{L}} := [0..L_1) \times \dots \times [0..L_{\mathfrak{D}})$ or, more gen., a finite connected subset $\mathbf{B} \subset \mathbb{Z}^{\times \mathfrak{D}}$. Some cells are occupied by pegs. A jump goes from $\boxed{\bullet \bullet \circ}$ to $\boxed{\circ \circ \bullet}$, and can be applied in all of the $2\mathfrak{D}$ directions. A **psn** (position) is a map $\Lambda: \mathbf{B} \rightarrow \{\bullet, \circ\}$; write $\Lambda \rightsquigarrow \Lambda'$ if one can jump from Λ to psn Λ' . For a cell $c \in \mathbf{B}$, let c^\bullet mean that c is occupied and all other cells are empty. For cells b, c , write $b \rightarrow c$ if $b^\circ \rightsquigarrow c^\bullet$. Say that b is **blocked** (w.r.t \mathbf{B}) if

there is no $c \in \mathbf{B}$ with $b \rightarrow c$. **a** (Use K4 for the Klein-4 argument.) On $\mathbb{Z}^{\times \mathfrak{D}}$, what are the K4-equiv-classes, and how many are there? What are the K4-blocked positions in $\mathbf{B}_{\vec{L}}$? In ${}^3+7$? In ${}^W+H$?

b Is \rightarrow symmetric? Is \rightarrow transitive? DTATTQuestions depend on the board?

c On $\mathbf{B} := 4 \times 4$, describe all pairs (b, c) st. $b \xrightarrow{K4} c$. Here, $\xrightarrow{K4}$ is the same as \rightarrow ? In $\mathbb{Z}^{\times \mathfrak{D}}$, two cells are **checker-equiv** if, in principle, checker could jump from one to the other. How many checker-equiv-classes are there? How can you

combine checker-equiv and K4 to find $b \rightarrow c$? **d** Generalize everything. E.g, a **coal**(escence) moves $\boxed{\bullet \bullet \bullet}$ to $\boxed{\circ \circ \circ}$. The K4 jump-invariant is also coal-invariant! On 4×4 , what is $\xrightarrow{\text{coal}}$ Triangle boards? Center-of-mass arguments?

X4: Pick a non-trivial problem from chapters 16–22, and solve it elegantly.

X1: ___ ___ ___ 120pts

X2: ___ ___ ___ 155pts

X3: ___ ___ ___ 165pts

X4: ___ ___ ___ 155pts

Total: ___ ___ ___ 595pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: _____

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