Staple!

Team: X

Calc III
MAC2313 3191

**X-Home**

Prof. JLF King
*Touch*: 30Sep2019

**Note.** The order of your hand-in should be: PROBLEM SHEET (this side up), NOTATION SHEET (if any), followed by the write-up(s) to each essay question.

General instructions are on the CHECKLIST. Page numbers without citation refer to our textbook. If you find an unfamiliar term, look in the "End Notes and Hints" section at the end of this problem-sheet. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797$\cdots$.Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

**X1:** Show no work. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word*.

___ a Let $\mathbf{G}(x,y,z) := x^2\widehat{\boldsymbol{\imath}} + xz\widehat{\boldsymbol{\jmath}} + y^2z\widehat{\mathbf{k}}$ and let $\mathbf{H} := \mathbf{Curl}[\mathbf{G}]$. Easily compute the following.

$\mathbf{H}=$ ⌞. . . . . ⌟;

$\mathbf{Curl}[\mathbf{H}]=$ ⌞. . . . . ⌟;

$\mathrm{Div}[\mathbf{H}]=$ ⌞. . . . . ⌟.

___ b* Let $C$ be the (oriented) arc of the unit circle which goes from the point $\frac{1}{2}[\widehat{\boldsymbol{\imath}} + \sqrt{3}\,\widehat{\boldsymbol{\jmath}}]$ counterclockwise to the point $-\widehat{\boldsymbol{\imath}}$. Compute the *path integral* of the vector field $\mathbf{G}(x,y) := xy^5\widehat{\boldsymbol{\imath}} + y\widehat{\boldsymbol{\jmath}}$.

$\int_C \langle \mathbf{G}, \mathrm{dLen}\rangle =$ ⌞. . . . . ⌟.

___ c Let $\mathbf{H}(x,y) := [2xy^3 + \cos(2x)]\widehat{\boldsymbol{\imath}} + [\mathsf{e}^{3y} + 3x^2y^2]\widehat{\boldsymbol{\jmath}}$ be a vector field. *Either* compute a potential function $f{:}\mathbb{R}^2{\to}\mathbb{R}$ so that $\nabla(f) = \mathbf{H}$ *or* write "DNE" to indicate that $\mathbf{H}$ is not a gradient v.f.

$f(x,y)=$ ⌞. . . . . ⌟.

___ d* Let $R$ be the region consisting of points $(x,y)$ in the first quadrant such that $x^4 + y^2 \leqslant 1$. By any means (hint, hint) compute the work done by the force-field $\mathbf{F}(x,y) := x\widehat{\boldsymbol{\imath}} + x^4y\widehat{\boldsymbol{\jmath}}$ in moving an object counterclockwise once around the boundary of $R$.

Work= ⌞. . . . . ⌟.

___ e A delicious doughnut (**DN**) just fits into a box of dimensions 4in by 4in by 1in. Using Thm of Pappus, how many cubic inches of delicious dough are needed to make this DN? Vol= ⌞. . . . . ⌟ $\mathsf{in}^3$.

How many $\mathsf{in}^2$ of glaze (yuck!) are needed to frost the doughnut? SurAr= ⌞. . . . . ⌟ $\mathsf{in}^2$.

**Essay questions. X2:** Consider a (solid) ball of radius 1, centered at the origin. Now discard the bottom half, leaving only the hemiball in the half-space $z \geqslant 0$. Fix a number $r \in (0,1)$ and drill a hole, of radius $r$, in the hemiball; the axis-of-symmetry of the hole is the axis-of-symmetry of the hemiball. Let $H$ denote the resulting "drilled hemiball". Let $\bar{z}(r)$ denote the distance of the *centroid* of $H$ from the origin (thus $\bar{z}(r)$ is $z$-coord of the centroid of $H$). From geometry alone, what do expect for these?:
$\lim_{r\searrow 0} \bar{z}(r)=$ ⌞. . . . . ⌟; $\lim_{r\nearrow 1} \bar{z}(r)=$ ⌞. . . . . ⌟. Computing,

$\bar{z}(r) =$ ⌞. . . . . ⌟.

**X3:** *Invent* (and, hopefully, solve) an interesting calculus/physics/geometry problem using ideas from vector fields, ballistics, centroids, Lagrange multipliers. As always, use grammatical English that would make Shakespeare weep with joy and admiration.

End of X-Home

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| **X1:** | ___ ___ ___ | 240pts |
| **X2:** | ___ ___ ___ | 100pts |
| **X3:** | ___ ___ ___ | 100pts |
| **Total:** | ___ ___ ___ | 440pts |

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ⌞. . . . . ⌟ ⌞ ⌟⌞ ⌟

Ord: ⌞. . . . . ⌟ ⌞ ⌟⌞ ⌟

Ord: ⌞. . . . . ⌟ ⌞ ⌟⌞ ⌟