Staple!

Team: ____________________

Calc III
MAC2313 3191

X-Home

Prof. JLF King
Touch: 1Feb2022

Please. Carefully adhere to the **Checklist**, and to the generic instructions. The order of your hand-in should be: PROBLEM SHEET (this side up), TYPOGRAPHY SHEET (if any), followed by the write-up(s) to each essay question.

X1: Show no work. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

___ ___ **a,b** Consider a (solid) ball of radius 1, centered at the origin. Now discard the bottom half, leaving only the hemiball in the half-space $z \geq 0$. Fix a number $r \in (0,1)$ and drill a hole, of radius r, in the hemiball; the axis-of-symmetry of the hole is the axis-of-symmetry of the hemiball. Let H denote the resulting "drilled hemiball". Let $\bar{z}(r)$ denote the distance of the *centroid* of H from the origin (thus $\bar{z}(r)$ is z-coord of the centroid of H). From $\boxed{\text{geometry alone}}$, what do expect for these?:
$\lim_{r\searrow 0} \bar{z}(r)=$ ________ ; $\lim_{r\nearrow 1} \bar{z}(r)=$ ________ . Computing,
$\bar{z}(r) =$ ________________________________ .

___ ___ **c** Let $\mathbf{G}(x,y,z) := x^2\widehat{\imath} + xz\widehat{\jmath} + y^2z\widehat{\mathbf{k}}$ and let $\mathbf{H} := \mathbf{Curl}[\mathbf{G}]$. Easily compute the following.
$\mathbf{H}=$ ________________________________ ;
$\mathbf{Curl}[\mathbf{H}]=$ ________________________________ ;
$\mathrm{Div}[\mathbf{H}]=$ ________________________________ .

___ ___ **d** Let C be the (oriented) arc of the unit circle which goes from the point $\frac{1}{2}[\widehat{\imath} + \sqrt{3}\,\widehat{\jmath}]$ counterclockwise to the point $-\widehat{\imath}$. Compute the *path integral* of the vector field $\mathbf{G}(x,y) := xy^5\widehat{\imath} + y\widehat{\jmath}$.
$\int_C \langle \mathbf{G}, \mathrm{dLen} \rangle=$ ________________________________ .

___ ___ **e** Let $\mathbf{H}(x,y) := [2xy^3 + \cos(2x)]\widehat{\imath} + [\mathrm{e}^{3y} + 3x^2y^2]\widehat{\jmath}$ be a vector field. *Either* compute a potential function $f{:}\mathbb{R}^2 \to \mathbb{R}$ so that $\nabla(f) = \mathbf{H}$ *or* write "DNE" to indicate that $\mathbf{H}$ is not conservative.

$f(x,y)=$ ________________________________ .

f Let R be the region consisting of points (x,y) in the first quadrant such that $x^4+y^2 \leq 1$. By any means (hint, hint) compute the work done by the force-field $\mathbf{F}(x,y) := x\widehat{\imath}+x^4y\widehat{\jmath}$ in moving an object counterclockwise once around the boundary of R.
Work= ________________________________ .

g Of $x^2+y^2+z^2 = 1^2$, let S be the upper hemisphere $z \geq 0$. Let $\mathbf{F} := y^2\widehat{\imath} + xy\widehat{\jmath} + xz\widehat{\mathbf{k}}$. Compute this surface integral [*Hint:* Stokes' thm] :
$\iint_S \langle \mathbf{Curl}_{\mathbf{F}}, \mathbf{n} \rangle \,\mathrm{dArea}=$ ________________________ .

h A delicious doughnut (**DN**) just fits into a box of dimensions 4in by 4in by 1in. Using Thm of Pappus, how many cubic inches of delicious dough are needed to make this DN? Vol= ________________________ in^3.

How many in^2 of glaze (yuck!) are needed to frost the doughnut? SurAr= ________________________ in^2.

X2:

Bonus: *Invent* (and, hopefully, solve) an interesting calculus/physics/geometry problem using ideas from vector fields, ballistics, centroids, Lagrange multipliers. As always, use grammatical English that would make Shakespeare weep with joy and admiration.

Notes and Hints. The Marston Science Library has mathematical dictionaries. Our Teaching Page has useful handouts. In parts of X1 or X2, polar coordinates may be useful. In X2, I suggest first *completely* understanding the $\theta=\frac{\pi}{2}$ case.

End of X-Home

X1:	___ ___ ___	240pts
X2:	___ ___ ___	110pts
Bonus:	___ ___	20pts
Total:	___ ___ ___	350pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: . . . .

Ord: . . . .

Ord: . . . .