Staple!

Team:_____

Calc III
MAC3474 3129

X-Home

Prof. JLF King
Touch: 6May2016

Please. The order of your hand-in should be: PROBLEM SHEET (this side up), TYPOGRAPHY SHEET (if any), followed by the write-up(s) to each essay question. **The staple should be vertical!** Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797···. Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

X1: Show no work.

__ a,b Consider a (solid) ball of radius 1, centered at the origin. Now discard the bottom half, leaving only the hemiball in the half-space $z \geqslant 0$. Fix a number $r \in (0,1)$ and drill a hole, of radius r, in the hemiball; the axis-of-symmetry of the hole is the axis-of-symmetry of the hemiball. Let H denote the resulting "drilled hemiball". Let $\bar{z}(r)$ denote the distance of the *centroid* of H from the origin (thus $\bar{z}(r)$ is z-coord of the centroid of H). From geometry alone, what do expect for these?:

$\lim_{r\searrow 0} \bar{z}(r) =$ ______ ; $\lim_{r\nearrow 1} \bar{z}(r) =$ ______ . Computing,

$\bar{z}(r) =$ ____________________ .

__ c Let $\mathbf{G}(x,y,z) := x^2\hat{\imath} + xz\hat{\jmath} + y^2z\hat{\mathbf{k}}$ and let $\mathbf{H} := \mathbf{Curl}_{\mathbf{G}}$. Easily compute the following.

$\mathbf{H}=$ ____________________ ;

$\mathbf{Curl}_{\mathbf{H}}=$ ____________________ ;

$\mathrm{Div}_{\mathbf{H}}=$ ____________________ .

__ d,e Let C be the (oriented) arc of the unit circle which goes from the point $\frac{1}{2}[\hat{\imath} + \sqrt{3}\,\hat{\jmath}]$ counterclockwise to the point $-\hat{\imath}$. Compute the *path integral* of the vector field $\mathbf{G}(x,y) := xy^5\hat{\imath} + y\hat{\jmath}$.

$\int_C \langle \mathbf{G}, \mathrm{dLen}\rangle =$ ____________________ .

__ f Let $\mathbf{H}(x,y) := [2xy^3 + \cos(2x)]\hat{\imath} + [\mathrm{e}^{3y} + 3x^2y^2]\hat{\jmath}$ be a vector field. *Either* compute a potential function $f:\mathbb{R}^2\to\mathbb{R}$ so that $\nabla(f) = \mathbf{H}$ *or* write "DNE" to indicate that $\mathbf{H}$ is not conservative.

$f(x,y)=$ ____________________ .

g,h Let R be the region consisting of points (x,y) in the first quadrant such that $x^4 + y^2 \leqslant 1$. By any means (hint, hint) compute the work done by the forcefield $\mathbf{F}(x,y) := x\hat{\imath} + x^4y\hat{\jmath}$ in moving an object counterclockwise once around the boundary of R.

Work= ____________________ .

i,j Compute the following iterated integral.

$$\int_0^{\pi/8}\int_0^{\cos(\theta)} r^4\sin(2\theta)\,\mathrm{d}x[r]\,\mathrm{d}x[\theta] = \_\_\_\_\_\_\_\_ .$$ 1

k A delicious doughnut (**DN**) just fits into a box of dimensions 4in by 4in by 1in. Using Thm of Pappus, how many cubic inches of delicious dough are needed to make this DN? Vol= __________ in^3.

How many in^2 of glaze (yuck!) are needed to frost the doughnut? SurAr= __________ in^2.

X2: ESSAY QUESTION: In $\mathbb{R}^3$, let $\mathbf{Y}_1$ and $\mathbf{Y}_2$ be (infinite) cylinders, each of radius 1, whose axes intersect; let $\theta \in (0, \frac{\pi}{2}]$ denote the angle between the axes. The intersection $\mathbf{Y}_1 \cap \mathbf{Y}_2$ comprises two closed curves; call the shorter curve S and the longer curve L. [Well...in the $\theta=\frac{\pi}{2}$ case, the curves are congruent.] Curves S and L intersect in two points; call one of these points P.

α For several values of θ, draw LARGE carefully labeled and shaded pictures of $\mathbf{Y}_1, \mathbf{Y}_2, S, L, P$. Certain cross-sections of the cylinders are useful; draw them too. From the geometry alone, without computation, predict how $\kappa_S(\theta)$ and $\kappa_L(\theta)$ [see below] will vary with θ. What should happen at $\theta = \frac{\pi}{2}$? ...as $\theta\searrow 0$? Etc.? Why? *Explain!*

β Give a rigorous proof that S is a *planar* curve. [This is NOT true if the axes miss each other, or if the cylinders have different radii. So your argument will perforce use these facts in some crucial way. (Think Symmetry.)]

γ In an appropriate coordinate system, give an equation for S. Ditto for L. Precisely what kind of curves are these, and *HDYKnow*?

δ At P, compute the **curvatures**

$\kappa_S(\theta)$= ....

and $\kappa_L(\theta)$= ....

of the two curves, *explaining* and *justifying* each step. You may quote, without proof, results from class, from the Teaching Page, and from handouts.

ε MAGIC! Suddenly cylinders $\mathbf{Y}_1$ and $\mathbf{Y}_2$ become **solid** cylinders; let $\mathbf{S} = \mathbf{S}(\theta)$ denote their solid of intersection. Compute its *surface area*:

$\text{SurAr}(\mathbf{S}(\theta))$= ....

Explain all the ideas on how you computed this. [*Hint:* There is a way to do this with very very little computation, because the surface is made up of pieces of a cylinder.]

Bonus: Invent, create, write a *clever* –preferably *funny*– mathematics story, perhaps using some real mathematics that we have learned. As always, use grammatical English that would make Shakespeare weep with joy and admiration.

Notes and Hints. The Marston Science Library has mathematical dictionaries. Our Teaching Page has useful handouts. In parts of X1 or X2, polar coordinates may be useful. In X2, I suggest first *completely* understanding the $\theta = \frac{\pi}{2}$ case.

End of X-Home

X1:	___ ___ ___	330pts
X2:	___ ___ ___	225pts
Bonus:	___ ___	25pts
Total:	___ ___ ___	555pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

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