Staple!

Team:_____

Sets and Logic MHF3202 8768 | **Home-X** | Prof. JLF King *Touch*: 4Oct2017

Due **Noon, Tues, 28Feb.**, slid *completely under* my office door, LIT402. Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. *In grammatical English* ***sentences***, TYPE *your essays on every* ***third*** *line* (usually), *so that I can easily write between the lines. Do* **not** *restate the question.* Essays violate the CHECKLIST at *Grade Peril...*

X1: On a 9×9 chessboard, 37 rooks are placed. Prove there exists a ***friendly*** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns.] [*Hint:* PHP] Illustrate the concepts in your proof with *large*, *useful* *Pictures*.

X2: For all natnums $k < n$, prove that $H_k \perp H_n$, where

$$H_k \quad := \quad 1 + 6^{[2^k]}.$$

[*Hint*: For each natnum m, define $G_m := -1 + 6^{[2^m]}$. Prove a divisibility relation among the Hs and the Gs, by induction. Then a common divisor of H_k and H_n must...]

Also, produce an index $\ell \in \mathbb{N}$ st. H_ℓ is not prime.

X3: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

___ a A seq $(L_n)_{n=4}^{\infty}$ is defined by $L_4 := 3$, $L_5 := 4$, and $\forall n \in [4\,..\,\infty)$: $L_{n+2} = L_{n+1} + L_n$. With $\alpha := \frac{1+\sqrt{5}}{2}$ and $\beta := \frac{1-\sqrt{5}}{2}$, then, $L_k = [P\cdot\alpha^k + Q\cdot\beta^k]$ for each $k \in [4\,..\,\infty)$, where $P=$ _____, $Q=$ _____.

___ b Compute the real $\alpha=$ _____ such that

$$3^{\alpha} \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \quad = \quad \sum_{j=0}^{1995} \binom{1995}{j} 8^j .$$

[*Hint:* The Binomial Theorem]

c The number of ways of picking 42 objects from 70 types is $\left[\!\!\left[{42 \atop 70} \right]\!\!\right]$ $\overset{\text{Binom}}{\underset{\text{coeff}}{=\!=}}$ (_____). And $\left[\!\!\left[{42 \atop 70} \right]\!\!\right] = \left[\!\!\left[{N \atop T} \right]\!\!\right]$, where $N=$ _____ $\neq 42$, and $T=$ _____.

End of Home-X

X1:	___ ___	95pts
X2:	___ ___	75pts
X3:	___ ___	80pts
Total:	___ ___ ___	250pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____