Staple!

Ord: ___ ___

Calc 3
MAC2313

Optional Project-X

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Touch: 4Aug2016

OYOP: Your 2 essay(s) must be TYPESET, and Double or Triple spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a ***new*** sheet of paper.

Due: By noon, on Friday, 07Dec2012, slid *completely* under my office door. Then please email me.

X1: For $m \in [1,\infty)$, let Γ_m denote the equi-angular spiral which crosses the x-axis at $Q_1 := \mathbf{(}1,0\mathbf{)}$ and, one wrap later, at $Q_m := \mathbf{(}m,0\mathbf{)}$, in *cartesian coords.* (When m=1, the "spiral" degenerates into a circle.) Let $\mathbf{P}_m = \big(\alpha_m, \beta_m\big)$ be the *cartesian coordinate* parametrization of Γ_m st. $\mathbf{P}_m(0) = Q_1$, $\mathbf{P}_m(2\boldsymbol{\pi}) = Q_m$ and $\mathbf{P}_m(t)$ wraps once whenever t increases by $2\boldsymbol{\pi}$. [After the second wrap, the spiral hits the x-axis at $\mathbf{(}m^2,0\mathbf{)}$.]

So $\alpha_m(t)=$ .....

& $\beta_m(t)=$ .....

___ a Drawing Good Pictures, compute that $L_m := \text{Length}\binom{Q_1 \smile Q_m}{\text{wrap}}$ is

$L_m=$ .....

Total-length of Γ_m going in to the origin from Q_1, is $T_m =$ .....

[*Hint:* As $m \searrow 1$, geometrically you expect $L_m \to ??$ and $T_m \to ??$; **do they?** (Think L'Hôpital.) As $m \nearrow \infty$, geometry tells you to expect $T_m \to ??$, and L_m to be asymptotic to ??. **Are they?**]

___ b Showing the interesting steps, compute from $\mathbf{F}()$ the arclength parametrization $\mathbf{A}(s) = \mathbf{(}x(s), y(s)\mathbf{)}$, of the spiral, satisfying that $\mathbf{A}(0) = \mathbf{F}(0)$. Indeed,

$x(s) =$ .....

___ c Create some *interesting* mathematical problem concerning these spirals. Elegantly solve the problem that you created, drawing nice pictures. *Show off!*

X2: For angles $\frac{\pi}{2} \geqslant \beta > \alpha \geqslant 0$, let $\mathbf{s}$ be the arc of the radius=1ft circle going from angle α to β. Let $f(\alpha,\beta) := \mathcal{V} + \mathcal{H}$, where $\mathcal{V}$ is the (area of the) region lying below $\mathbf{s}$ and above the x-axis, and $\mathcal{H}$ is the region horizontally between $\mathbf{s}$ and the y-axis. Integrating,

$f(\alpha,\beta)=$ ..... $\cdot \text{ft}^2$. [*Hint:* Slice $\mathcal{V}$ vertically and $\mathcal{H}$ hor., integrating w.r.t θ. What amazing thing happens?]

End of *Optional* Project-X

X1:	___ ___ ___	155pts
X2:	___ ___ ___	105pts
Total:	___ ___ ___	260pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: .....