

X5: Short answer: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. Use ‘Rv’ for “random variable”, and ‘Var’ for “variance”.

 Circle	The stmt " $\forall N$ there exists a prime $p \in [N..2N]$ " is called:
Prime Number Thm	Riemann Hypothesis
Eratosthenes' Wild Guess	Merten's Thm
Bertrand's Postulate	

[a] For $|x| < 1$, use the “ $xddx$ trick” to compute

$$\sum_{n=0}^{\infty} nx^n = \quad \text{& } \sum_{n=0}^{\infty} n^2 x^n =$$

b Rv X is the number of flips till first head, where $P(\text{heads})=\frac{1}{5}$. So $E(X)=$... ↴. And $E(X^2)=$ ↴. Hence $\text{Var}(X)=$.

 The discriminant of poly $h(z) := z^2 - z + 3$ is $B^2 - 4AC = \underline{\dots}$. Let α and β be h 's roots. Set $\zeta := 4 + \alpha$. Its norm is $\mathcal{N}_h(\zeta) = \underline{\dots} \in \mathbb{Z}$. Its h -conjugate $\bar{\zeta} = \underline{\dots} + \underline{\dots} \alpha$. Finally, $[1/\zeta] = \underline{\dots} + \underline{\dots} \alpha$. [All coeffs are rational.]

d For posint N , let $\lambda(N)$ be the *smallest* $J \in \mathbb{Z}_+$ with $\forall x \in \Phi(N) : x^J \equiv_N 1$. Then

$\lambda(3) = \text{_____}$, $\lambda(5) = \text{_____}$ and $\lambda(15) = \text{_____}$.

Given posints K and L , write $\lambda(3^K \cdot 5^L)$ as product of prime-powers:

Essay question: Write in complete sentences and also fill-in the blanks. Each essay starts a new page.

X6: Let $N := 561 \stackrel{\text{note}}{=} 3 \cdot 11 \cdot 17$. Note

*: For each prime $p \nmid N$: $p-1 \mid N-1$.

Prove: $(\text{For each } k \perp N: k^{N-1} \equiv_N 1)$.

[Hint: Chinese Remainder Thm and Fermat's Little Thm.]

Bonus: The 14th cyclotomic polynomial is

$$\mathbf{C}_{14}(z) =$$

End of Class-X

X-Home: 365pts

X5: 120pts

X6: 65pts

Bonus: 10pts

Total: 550pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: