Sets and Logic
MHF3202 8768

Class-X

Prof. JLF King
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X4: Short answer. Show no work.
Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.
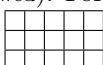
a Sequence $\overrightarrow{\mathbf{L}} := (L_n)_{n=0}^{\infty}$ is defined by $L_0 := 3$, $L_1 := 11$, and $\boxed{\forall n \in \mathbb{N}: \ L_{n+2} = -L_{n+1} + 6L_n}$. This implies $\boxed{\forall k \in \mathbb{N}: \ L_k = [P\cdot\alpha^k + Q\cdot\beta^k]}$, for real numbers

$\alpha=$ �� $> \beta=$ �� , $P=$ �� , $Q=$ �� .

b The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 5 atoms, so I have [expressed as single integer] �� many possibilities.

c The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is �� .

OYOP: *In grammatical English* **sentences**, *write your essays on every* **third** *line* (usually)*, so that I can easily write between the lines. Do* **not** *restate the question.* Start each essay on a <u>new</u> sheet-of-paper. Put ordinal also on back of last page, **large**, right-side-up.

X5: An ***Lmino*** (pron. "ell-mino") comprises three squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{B}_N$ denote the $3{\times}N$ board: I.e, is the $\mathbf{B}_5$ board. Prove:

> Theorem: When N is odd, then board $\mathbf{B}_N$ is <u>not</u> Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, <u>*labeled*</u> pictures.

X6:
Over a 29 day month, SeLoidian Bubba posts at least one soln per day, for a total of 45 solns. PROVE:

> There is a period of consecutive days over which he posted exactly 12 solutions.

NOTE: In your proof, let s_n denote the number of solns posted that month by the end of day n. By hyp., then,

$$1 \leqslant s_1 < s_2 < \ldots < s_{29} = 45.$$

Let $t_n := 12 + s_n$. *Using this notation, write a complete, rigorous proof, proving any lemmas you need/want.*

<div style="border:1px solid blue; text-align:center; color:blue">End of Class-X</div>