Staple!

Ord: ___ ___

Plex
MAA4402 8436

Class-X

Prof. JLF King
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Read! the Notation! Use $\mathsf{C}_r := \mathrm{Sph}_r(0)$.

The expression $\textsc{Function} = \mathsf{u} + \mathbf{i}\mathsf{v}$ means that u is the real-part of $\textsc{Function}$, and v is the imaginary-part.

$\mathrm{Res}(\beta, \mathbf{q})$ is the residue of *function* $\beta()$ at the point $\mathbf{q}$. Please simplify answers, when possible.

X1: Short answer. Show no work.

20 **a** Which action *loses* pts? (circle) *Writing-in-sentences* *Writing-t-different-from-$+$* *Tiny - tiny - writing* *Writing-LARGE*

10 25 **b** Fnc $\alpha(z) := \dfrac{\cosh(7z)}{[z-2]^2\, z^2}$ has a $z{=}0$ pole of order ____.
And $\mathrm{Res}(\alpha, 0) =$ ____.

25 **c** Let $J := \displaystyle\int_{\mathsf{C}_2} z^4 \sin(\tfrac{7}{z})\, \mathrm{d}z$. So $J =$ ____

10 25 **d** Defining $\vec{\mathbf{a}} = (a_0, a_1, \dots)$ by $\cos(2z) = \sum_{n=0}^{\infty} a_n z^n$, coefficient $a_4 =$ ____. Let

$$\sum_{n=0}^{\infty} r_n z^n = \frac{1}{\cos(2z)}$$

be the reciprocal PS. Then $r_4 =$ ____.

[*Hint:* You can compute the reciprocal as we did in class.]

X2: Short answer. Show *no* work.

Let $f(z) := z^4 + 5z^2 + 4$. Reciprocal $H(z) := 1/f(z)$ has, in the upper half-plane, two poles $\mathbf{p}$ and $\mathbf{q}$, where $\mathbf{p}$ lies closer to the origin than $\mathbf{q}$.

So $\mathrm{Res}(H, \mathbf{p}) =$ ____ *and* $\mathrm{Res}(H, \mathbf{q}) =$ ____.

Our D-contour technique applies to H.

Thus $J := \displaystyle\int_{-\infty}^{+\infty} \frac{1}{z^4 + 5z^2 + 4}\, \mathrm{d}x =$ ____.

OYOP: *In grammatical English* ***sentences****, write your essay on every 2*^nd^ *line* (usually)*, so I can easily write between the lines.*

X3: **α** Precisely state the Gauss Mean-value thm, with all the hypotheses. State it as a *formal* theorem.

β Carefully prove the Gauss MVT, using the CIF.

X4: Consider an entire fnc $g = \mathsf{u} + \mathbf{i}\mathsf{v}$ whose imaginary-part, v is bounded, e.g, $|\mathsf{v}(z)| \le 5$ for all z. Prove that g is constant. [*Hint:* Somehow prove that g is bounded, then apply SomebodyOrOther's thm. (*Who's thm?* State the theorem formally.) You may want to construct an auxiliary function from g.]

X1:	___ ___ ___	115pts
X2:	___ ___	60pts
X3:	___ ___	55pts
X4:	___ ___	40pts
Total:	___ ___ ___	270pts

Please PRINT your name *and* ordinal*. Ta:*

____ Ord:

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____