Differential Eqns
MAP2302

X-Class

Prof. JLF King
Tuesday, 20Apr2021

Note. Write unambiguously e.g, $1/a + b$ should be *bracketed* either $[1/a] + b$ or $1/[a + b]$, as appropriate. (Be careful with **negative** signs!)

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ **or** $[\sin x]$.

X1: *Short answer. Show no work.*

a 10 When possible, Prof. King thinks it is beneficial to *estimate* an answer, via an *independent* method, before computing. *T* *F*.

b 10 10 10 Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$\begin{aligned} x' + \ x - 3y &= 0\,, \\ y' + 6x - 8y &= 0\,. \end{aligned}$$

It can be written as $\mathsf{Y}' = \mathsf{M}\cdot\mathsf{Y}$,
where $\mathsf{Y} := \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ and M is matrix 5

You may write a $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *matrix as* $[a, b\,;\, c, d]$.

Characteristic poly of M is $\wp_{\mathsf{M}}(z)=$ ....

A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for *numbers* $\alpha=$ .... and $\beta=$ ...., where $\alpha \leqslant \beta$.

c 5 10 10 Gamma fnc: $\mathbf{\Gamma}(7)=$ .... and $\mathbf{\Gamma}(\frac{7}{2})=$ ....

For all real $x > 1$, our $\mathbf{\Gamma}()$ function satisfies recurrence relation

$\mathbf{\Gamma}(x) =$ ....

d 10 10 $[x^3 \circledast x^2]=$ .... . $[x^K \circledast x^N]=$ ....

e 30 Suppose $y(0) = 2$, $y'(0) = 3$, $y''(0) = 5$. Then $\mathcal{L}\big(y^{(3)} + 2y'\big)(s)$ equals $\big[[p(s)\cdot\widehat{y}(s)] + q(s)\big]$ for **polynomials**

$p(s)=$ ....

and $q(s)=$ ....

f 30 Let $H(t) := \int_0^t \cos(2\cdot[t-w])\cdot e^{5w}\,dw$. Its Laplace transform is $\widehat{H}(s) = \frac{Q(s)}{R(s)}$, where *polynomials*
$Q(s) =$ .... $R(s) =$ ....
have no common factor.

[*Hint:* It is unnecessary to compute the integral. A polynomial may be written as a product; you do not need to multiply out.]

X2: *"I have neither requested nor received help on this exam other than from my professor."* 5

X1:	___ ___ ___	145pts
X2:	___	5pts
Total:	___ ___ ___	150pts