Staple!

Team: ____

Sets and Logic MHF3202 8768 — **Home-W** — Prof. JLF King, *Touch:* 6May2016

Due **BoC, Mon, 06Feb2012**, Please *fill-in* every *blank* on this sheet. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

For the two essay questions, carefully TYPE *triple–spaced, grammatical, solutions. I suggest* LaTeX. *Essays violate the* CHECKLIST *at Grade Peril...*

W1: Let $L(k) := [5^{[2k]}] - 1$. By induction on k, prove that $\forall k \in \mathbb{N}:\ L(k) \bullet 3$.

W2: Interval-of-integers $\mathbf{J} := [101\,..\,200)$ has 99 elements. A subset $S \subset \mathbf{J}$ is ***Big*** if $|S| = 51$. Subset $S \subset \mathbf{J}$ is ***Perfect*** if there exist *distinct* members $x,y \in S$ st. $x + y = 300$.

Prove that Big $\Rightarrow$ Perfect. [*Hint:* PHP. *Carefully* specify what your pigeon-holes are.]

W3: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

___ ___ **a** Define $G{:}[1\,..\,12]\circlearrowleft$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is ________. The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ________________.

___ ___ **b** LBolt: $\text{Gcd}(70, 42) =$ ______ $\cdot 70 +$ ______ $\cdot 42$.

So (LBolt again) $G := \text{Gcd}(70, 42, 30) =$ ______ and ______ $\cdot 70 +$ ______ $\cdot 42 +$ ______ $\cdot 30 = G$.

___ ___ **c** As polynomials in $\Gamma := \mathbb{Z}_{11}[x]$, let

$$B(x) := x^4 - 4x^3 + 2x^2 - 3x - 2\,;$$
$$C(x) := 2x^3 - 5x + 5\,.$$

Write t.fol polys, using coeffs in [-5 .. 5]; use $\equiv$ for equality in $\mathbb{Z}_{11}$ and in Γ. Compute quotient and remainder polys, $q(x) \equiv$ ______ & $r(x) \equiv$ ______, with $B \equiv [q \cdot C] + r$ and $\text{Deg}(r) < \text{Deg}(C)$.

Let $D := \text{Gcd}(B, C)$. **Monic** $D(x) \equiv$ ______.

Compute polys $S(x) \equiv$ ______, $T(x) \equiv$ ______ st. $[S \cdot B] + [T \cdot C] \equiv D$.

d $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. *T* *F*

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. *T* *F*

For all sets Ω, *there exists a fnc* $f{:}\mathbb{R}\to\Omega$. *T* *F*

End of Home-W

W1:	___ ___	65pts
W2:	___ ___	65pts
W3:	___ ___ ___	120pts
Total:	___ ___ ___	250pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* Name/Signature/Ord

Ord: ________________

Ord: ________________

Ord: ________________