Staple!

Team W_____

Sets and Logic MHF3202 8768 | **Home-W** | Prof. JLF King *Touch*: 4Oct2017

Due **BoC, Monday, 10Feb2014**, Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. *In grammatical English* ***sentences***, TYPE *your essays on every* ***third*** *line* (usually), *so that I can easily write between the lines. Do* **not** *restate the question.*

W1: On a 9×9 chessboard, 37 rooks are placed. Prove there exists a ***friendly*** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns.] [*Hint:* PHP] Illustrate the concepts in your proof with *large*, *useful* *Pictures*.

W2: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \dots)$ by $b_0 := 0$ and $b_1 := 3$ and

†: $$b_{n+2} := 7b_{n+1} - 10b_n\,, \quad \text{for } n = 0, 1, \dots .$$

Use induction to prove, for each natnum k, that

‡: $$b_k = 5^k - 2^k\,.$$

Further: Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $\boxed{b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n}$, for arbitrary real-number coefficients **S** and **P**?

W3: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

___ a The number of permutations of "PREPPER" is _____.

___ b The coeff of x^7y^{12} in $[5x + y^3 + 1]^{30}$ is _____.

___ c $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. T F
$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. T F
For all sets Ω, *there exists a fnc* $f:\mathbb{R}\to\Omega$. T F

___ d Compute the real $\alpha =$ _____ such that

$$3^{\alpha} \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \;=\; \sum_{j=0}^{1995} \binom{1995}{j} 8^j\,.$$

[*Hint:* The Binomial Theorem]

___ e The number of ways of picking 42 objects from 70 types is $\left[\!\!\left[{42 \atop 70} \right]\!\!\right] \underset{\text{coeff}}{\overset{\text{Binom}}{=}} \left(\quad\right)$. And $\left[\!\!\left[{42 \atop 70} \right]\!\!\right] = \left[\!\!\left[{N \atop T} \right]\!\!\right]$, where $N =$ _____ $\neq 42$, and $T =$ _____.

End of Home-W

W1:	___ ___	95pts
W2:	___ ___	75pts
W3:	___ ___	95pts
Ouch!, scratch work handed-in; OR Poorly stapled. :	___ ___	−20pts
Total:	___ ___ ___	265pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____