Number Theory 2
MAT4930 5662

Class-W

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W5: Short answer: <u>Show no work</u>. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

[z] *Is 2 prime?* ⬚Circle⬚ the best response:

Gimme a break! 2 is even!

Whoa, dude. Where's my calculator?!

[a] $\varphi(18)=$ ___. Let $\langle\cdot\rangle_N$ mean the non-negative mod-N residue. So $\langle 5^{122}\rangle_{18}=$ ___. Thus $\langle 2^{5^{122}}\rangle_{19}=$ ___.

[b] Writing 134! in decimal, it ends (on the right) with ___ zeros.

[c] Let $\mathbf{C}_n$ denote the nth cyclotomic poly. Then $\mathbf{C}_4(z)=$ ___ and $\text{Deg}(\mathbf{C}_{100})=$ ___.

Essay questions: Write in complete sentences and <u>also</u> fill-in the blanks. Each essay starts a new page.

W6: The Riemann zeta fnc is $\zeta(s)=$ ___.

This converges for all s ___.

Justify the Euler's Identity, $\zeta(s) = \prod_p \left[1 - p^{-s}\right]^{-1}$, by the "Sieve-of-Eratosthenes method" that we did in class.

W7: Prove the PowFour Lemma,

For each posint N: $\left[\prod_{p \leqslant N} p\right] < 4^N$,

by the induction argument we did in class.

Bonus: In the general population, 2% have disease, X. There is a test for X, with a 13% false-positive rate, and a 3% false-negative. Dave tests positive for X. Bayes' theorem says that the probability that Dave actually has the disease is ___.

(Please express your answer ITForm $\frac{a\cdot b}{c\cdot d \ + \ e\cdot f}$ of BT. Assume this independence: 2% *of-those-tested* have X.)

⬚ End of Class-W ⬚

Please PRINT your Name

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