Staple!

Ord: ___ ___

Sets and Logic
MHF3202 09EH

Class-W

Prof. JLF King
Wednesday, 20Nov2019

W1: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word*.

a A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is two courses. Circle:

Yes **True** **Darn tootin'!**

b Let $T(m) := \frac{1}{2}m[m+1]$ be the m^{th} triangular number. An explicit bijection $F: \mathbb{N}\times\mathbb{N} \hookrightarrow\!\!\!\!\twoheadrightarrow [-4..\infty)$ is
$F(n,k) :=$ ....

c A "Cantor's-Hotel" type bijection $f:(5,6] \hookrightarrow\!\!\!\!\twoheadrightarrow (0,1)$ is:
$f($ .... $) :=$ .... , for *each* posint n;
and $f(x) :=$ .... , for *each* $x \in (5,6] \smallsetminus C$,
where $C :=$ ....

d *On $\mathbb{R}_+$, define several relations: Say that $x\mathcal{R}y$ IFF $y - x < 17$. Define $\mathcal{P}$ by: $x\mathcal{P}y$ IFF $x^{\log(y)} = 5$.*
Say that $x\mathcal{I}y$ IFF $x + y$ is irrational.
Use $\bullet|$ for the "divides" relation on the positive integers: $k \bullet| n$ iff there exists a posint r with $rk = n$.

d_1 Please circle those of the following relations which are *transitive* (on their domain of defn).

$\neq$ $\bullet|$ $\leqslant$ $\mathcal{R}$ $\mathcal{P}$ $\mathcal{I}$

d_2 Circle the *symmetric* relations:

$\neq$ $\bullet|$ $\leqslant$ $\mathcal{R}$ $\mathcal{P}$ $\mathcal{I}$

d_3 Circle the *reflexive* relations:

$\neq$ $\bullet|$ $\leqslant$ $\mathcal{R}$ $\mathcal{P}$ $\mathcal{I}$

e Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite.
Stmt "*This $\approx$ is an equivalence-relation*" is: *T* *F*

f The number of permutations of "PREPPER", as a multinomial coefficient, is .... $\overset{\text{as a number}}{=}$ ....

OYOP: *In grammatical English sentences, write your essay on every 2nd line (usually), so that I can easily write between the lines.*

W2: Between sets $\mathbf{A} := \mathbb{Z}_+$ and $\boldsymbol{\Omega} := \mathbb{N}$, consider injections $f: \mathbf{A} \hookrightarrow \boldsymbol{\Omega}$ and $g: \boldsymbol{\Omega} \hookrightarrow \mathbf{A}$, defined by

$$f(z) := 3z \quad \text{and} \quad g(\beta) := \beta + 5\,.$$

The S-B thm produces a set $Y \subset g(\boldsymbol{\Omega}) \subset \mathbf{A}$ so that, letting $X := \mathbf{A} \smallsetminus Y$, function $\theta: \mathbf{A} \hookrightarrow\!\!\!\!\twoheadrightarrow \boldsymbol{\Omega}$ is a *bijection*, where

$$*: \qquad \theta\rfloor_X := f\rfloor_X \quad \text{and} \quad \theta\rfloor_Y := g^{-1}\rfloor_Y\,.$$

i Prove, for these particular injections, that there is only *one* set Y which makes $(*)$ a bijection. [Does an orbit-picture help?]

ii Compute $\theta(56) =$ .... and $\theta(83) =$ .... , drawing the appropriate part of (f,g)-orbit pictures.

W1:	___ ___ ___	135pts
W2:	___ ___	55pts
Total:	___ ___ ___	190pts

NAME: .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....