Staple!

Sets and Logic MHF3202 8768	**Class-W**	Prof. JLF King 6Sep2015

End of Class-W

W1:	___ ___	45pts
W2:	___ ___	90pts
Total:	___ ___ ___	135pts

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines. Do* **not** *restate the question.*

W1: Prop. Q_k: *There are at least k prime numbers.*

Prove, for $k \in \mathbb{Z}_+$, that $(Q_k) \Rightarrow (Q_{k+1})$. For free, you may use that each posint factors as a product of primes. [*Hint:* You likely want to state and prove a coprimeness lemma first.]

W2: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Define fncs $G,P:[1\,..\,12]\circlearrowleft$, where $G(n)$ is the number of letters in the n^{th} Gregorian month [so $G(2) = 8$; the 2^{nd} month "February" has 8 letters], and $P(n) := 13 - n$.

The *set* of posints k with $G^{\circ k}(1) = G^{\circ k}(2)$ is ______.

Let $f := P \circ G$. Then $f^{\circ 2}(11)=$ ______.

b Mod K:=229, the recipr. $\langle \frac{1}{45} \rangle_K =$ ______ $\in [0\,..\,K)$.

[*Hint:* $\frac{1}{4}$] So $x=$ ______ $\in [0\,..\,K)$ solves $5 - 45x \equiv_K 8$.

c Posints $L=$ ___, $N=$ ___, $\alpha=$ ___, $\beta=$ ___, are st. $\alpha \equiv_L \beta$, yet $N^\alpha=$ ___ is **not** $\equiv_L$ to $N^\beta=$ ___.

d As polynomials in $\Gamma := \mathbb{Z}_7[x]$, let

$$B(x) := x^4 - 2x^3 + x - 2\,;$$
$$C(x) := x^3 + 3x^2 - 3x\,.$$

Write t.fol polys, using coeffs in [-3 .. 3]; use $\equiv$ for equality in $\mathbb{Z}_7$ and in Γ. Compute quotient and remainder polys, $q(x) \equiv$ ______ & $r(x) \equiv$ ______, with $B \equiv [q \cdot C] + r$ and $\text{Deg}(r) < \text{Deg}(C)$.

Let $D := \text{Gcd}(B, C)$. **Monic** $D(x) \equiv$ ______.

Compute polys $S(x) \equiv$ ______, $T(x) \equiv$ ______ st. $[S \cdot B] + [T \cdot C] \equiv D$.

e Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\ f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: ______. $E2$: ______. $E3$: ______.