Staple!

Sets and Logic
MHF3202 2787

Class-W

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OYOP: For your 2 Essays: *Write your grammatical English sentences on every third line, so that I can easily write between the lines. Start each essay on a new sheet of paper.*

W1: Let $\mathbf{J} := [0,1]$. You may use, without proof, the Schröder-Bernstein thm and the following.

$\mathbf{a_1}$: $\mathbb{R} \asymp \{0,1\}^{\mathbb{N}}$. $\mathbf{a_2}$: $\mathbb{N}\times\mathbb{R} \asymp \mathbb{R}$.

$\mathbf{a_3}$: *For each three sets* Ω, B, D: $\Omega^{B\times D} \asymp [\Omega^B]^D$.

$\mathbf{a_4}$: *The set* $S := \mathbb{Q}\cap\mathbf{J}$ *is countable.*

Prove that $\mathbf{C}(\mathbf{J})$, *the set of continuous functions* $\mathbf{J}\to\mathbb{R}$, *is bijective with* $\mathbb{R}$. Cite each $(\mathbf{a}_i)$ where you use it. Specify what Ω, B, D are, when you apply $(\mathbf{a_3})$. [*Note:* Does your proof split into easily-understood lemmas?]

W2: Between sets $\mathbf{A} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $f\colon\mathbf{A}\hookrightarrow\mathbf{\Omega}$ and $g\colon\mathbf{\Omega}\hookrightarrow\mathbf{A}$, defined by

$$f(z) := 3z \quad\text{and}\quad g(\beta) := \beta+5\,.$$

The S-B thm produces a set $Y \subset g(\mathbf{\Omega}) \subset \mathbf{A}$ so that, letting $X := \mathbf{A}\smallsetminus Y$, function $\theta\colon\mathbf{A}\hookrightarrow\!\!\!\rightarrow\mathbf{\Omega}$ is a *bijection*, where

$$*:\qquad \theta\rfloor_X := f\rfloor_X \quad\text{and}\quad \theta\rfloor_Y := g^{-1}\rfloor_Y\,.$$

i Prove, for these particular injections, that there is only *one* set Y which makes $(*)$ a bijection. [Does an orbit-picture help?]

ii Compute $\theta(56)=$ ________ and $\theta(83)=$ ________, drawing the appropriate part of (f,g)-orbit pictures.

W3: Short answer. Show no work.
Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Repeating decimal $0.3\overline{12}$ equals $\frac{n}{d}$, where posints $n\perp d$ are $n=$ ________ and $d=$ ________.

b An explicit bijection $F\colon\mathbb{N}\times\mathbb{N}\hookrightarrow\!\!\!\rightarrow[-4\,..\,\infty)$ is $F(n,k) :=$ ________________.

c Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A\approx B$ IFF $A\cap B$ is infinite. Stmt "*This* $\approx$ *is an equivalence-relation*" is: *T* *F*

d Each three sets Ω, B, C engender a natural bijection, $\Theta\colon\Omega^{B\times C}\hookrightarrow\!\!\!\rightarrow[\Omega^B]^C$, defined, for each $f\in\Omega^{B\times C}$, by

$\Theta(f) := \Big[c\mapsto[$ ________________ $]\Big]$.

Its inverse-map $\Upsilon\colon[\Omega^B]^C\hookrightarrow\!\!\!\rightarrow\Omega^{B\times C}$ has, for $g\in[\Omega^B]^C$, $\Upsilon(g) := \Big[(b,c)\mapsto[$ ________________ $]\Big]$.

End of Class-W

W1:	___ ___	75pts
W2:	___ ___	60pts
W3:	___ ___ ___	110pts
Total:	___ ___ ___	245pts