Staple!

Ord: ___ ___

Plex
MAA4402 8436

Class-W

Prof. JLF King
Wedn, 27Oct2021

Welcome! Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

Write unambiguously e.g, $1/a+b$ should be *bracketed* either $[1/a]+b$ or $1/[a+b]$, as appropriate. (Be careful with **negative** signs!) Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

W1: Short answer. Show no work.

a (10) Prof. King wears bifocals, and cannot read small handwriting. Circle one: **True!** **Yes!** ***Who??***

b (15) Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, $500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is Sunday, March 27, 2022.

Circle: **Yes** **True** ***Résumé material!***

c (25) Define $f(x+\mathbf{i}y) := xy + \mathbf{i}x$. Let L be the line-segment from the origin to $2+\mathbf{i}$. Then $\int_{\mathsf{L}} f(z)\,dz =$ ____.

d (30) Let C be SCC $\mathrm{Sph}_3(\mathbf{i})$, a circle of radius 3. Value $\oint_{\mathsf{C}} \frac{e^{3z}}{[z-2]^5}\,dz =$ ____

[Answer may be written as a product, using powers and factorials.]

e (30) Fnc $\mathsf{u}(x,y) := 2xy + x$ has harmonic conjugate $\mathsf{v}(x,y)=$ ____.

W2: Short answer; fill-in the 5 blanks.

Consider the logarithmic spiral, S, turning CCW as it spirals out to ∞, crossing the $\mathbb{R}$-axis at points $\{3^n\}_{n\in\mathbb{Z}}$.

i Similar to what we did in class, naturally parametrize S by $\sigma:\mathbb{R}\to\mathbb{C}$ of form $\sigma(t) = e^{M\cdot t}$, where $M =$ ____, with $\sigma(-2\pi) = \frac{1}{3}$, $\sigma(0) = 1$, $\sigma(2\pi) = 3, \ldots, \sigma(2\pi\cdot n) = 3^n$.

ii Let **A** be the *Arclength* of one turn of the spiral, starting at $3\in\mathbb{R}$, ending at $9\in\mathbb{R}$. Set up the specific integral $\mathbf{A} = \int_\alpha^\beta f(t)\,dt$, where $f(t)=$ ____, with $\alpha=$ ____ and $\beta=$ ____.

iii Computing the integral gives number **A**= ____.

OYOP: *In grammatical English* ***sentences****, write your essay on every* 2^{nd} *line* (usually)*, so I can easily write between the lines.*

W3: Below, $h:\mathbb{C}\to\mathbb{C}$, and S $\subset\mathbb{C}$ is a closed-curve, and $\mathbf{w}\in\mathbb{C}$ is an *appropriate* point.

α Detailing the precise conditions needed on h, S and $\mathbf{w}$, *carefully* state the Cauchy Integral Formula Theorem.

β Recall the Cauchy Homotopy Thm: *Suppose closed-curves* S *and* R *are homotopic in an open set on which a fnc* f *is holomorphic. Then* $\oint_{\mathsf{S}} f = \oint_{\mathsf{R}} f$.

Use the above CHT to give a formal proof of the Cauchy Integral Formula Theorem. Also draw LARGE pictures showing the ideas in the proof.

W1:	___ ___ ___	110pts
W2:	___ ___	65pts
W3:	___ ___	85pts
Total:	___ ___ ___	260pts

Please PRINT your name and ordinal. Ta:

Ord: ____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____