

Linear Algebra MAS4105 14G8 Class-W Prof. JLF King Tuesday, 05Apr2022

**Note.** The geometry on  $\mathbb{R}^n$  is defined by the dot-product.  
Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE**  $\neq \{\}$   $\neq 0$ .

**W1:** Show no work.

**a** For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course Circle:

Yes True Darn tootin'!

**b** In  $\mathbb{R}^3$ , the closest point to  $\mathbf{v} := (1, -7, -5)$  on the line through  $\mathbf{0}$  and  $\mathbf{q} := (1, 2, 3)$ , is  $(\text{.....}, \text{.....}, \text{.....})$ .

In  $\mathbb{R}^2$ , with  $\mathbf{s} := (1, 8)$  and  $\mathbf{w} := (4, -2)$ , compute  $\text{Orth}_{\mathbf{w}}(\mathbf{s}) = \text{.....}$ .

**c** Put inner-product  $\langle f, g \rangle := \int_0^2 [f \cdot g]$  on the VS of polynomials. With  $\mathbf{D} := 1 + x$  and  $\mathbf{u} := x^2$ , compute  $\text{Proj}_{1+x}(x^2) = \text{.....}$ .

**d** Let  $R_\theta$  be the std. rotation [by  $\theta$ ] matrix. With

$$C := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad B := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product  $[CB]^{35} = \alpha \cdot R_\theta$ , with  $\alpha = \text{.....} \in \mathbb{R}_+$   
and  $\theta = \text{.....} \in (-180^\circ, 180^\circ]$ .

**e**  $\mu = \text{.....} \leq \nu = \text{.....}$

are the eigenvals of  $G := \begin{bmatrix} 11 & 30 \\ -6 & -16 \end{bmatrix}$ . Let  $D := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$ .  
Then  $D = U^{-1}GU$  where the  $2 \times 2$  integer matrix  $U$  is

$$U = \left[ \begin{array}{c|c} & \\ \hline & \end{array} \right].$$

OYOP: Essay: *Write on every **second** line, so that I can easily write between the lines.*

**W2:** Matrix  $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ , where  $A$  and  $D$  are  $5 \times 5$  and  $7 \times 7$ , resp. Suppose  $C$  is the  $7 \times 5$  **zero-matrix**. Prove that  $\text{Det}(M) = \text{Det}(A) \cdot \text{Det}(D)$ . [Hint: A good picture helps.]

**Before** starting your proof, state precisely the formula for determinant that you are using.

End of Class-W

**W1:** \_\_\_\_ 165pts

**W2:** \_\_\_\_ 65pts

**Total:** \_\_\_\_ 230pts

NAME: \_\_\_\_\_

**HONOR CODE:** *"I have neither requested nor received help on this exam other than from my professor."*

Signature: \_\_\_\_\_