

Note. The geometry on \mathbb{R}^n is defined by the dot-product.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

W1: Show no work.

15 For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course Circle:

Yes

True

Darn tootin'!

40 **b** In \mathbb{R}^3 , the closest point to $\mathbf{v} := (1, -7, -5)$ on the line through $\mathbf{0}$ and $\mathbf{q} := (1, 2, 3)$, is $(\underline{\dots}, \underline{\dots}, \underline{\dots})$.

In \mathbb{R}^2 , with $\mathbf{s} := (1, 8)$ and $\mathbf{w} := (4, -2)$, compute $\text{Orth}_{\mathbf{w}}(\mathbf{s}) =$

30 **c** Put inner-product $\langle f, g \rangle := \int_0^2 [f \cdot g]$ on the VS of polynomials. With $\mathbf{D} := 1 + x$ and $\mathbf{u} := x^2$, compute $\text{Proj}_{1+x}(x^2) =$

40 **d** Let \mathbf{R}_θ be the std. rotation [by θ] matrix. With

$$\mathbf{C} := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad \mathbf{B} := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product $[\mathbf{CB}]^{35} = \alpha \cdot \mathbf{R}_\theta$, with $\alpha =$

and $\theta = \underline{\dots} \in (-180^\circ, 180^\circ]$.

10 10 10 10 **e** $\mu =$

$$\leq \nu =$$

are the eigenvals of $\mathbf{G} := \begin{bmatrix} 11 & 30 \\ -6 & -16 \end{bmatrix}$. Let $\mathbf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$.

Then $\mathbf{D} = \mathbf{U}^{-1} \mathbf{G} \mathbf{U}$ where the 2×2 integer matrix \mathbf{U} is

$$\mathbf{U} = \left[\begin{array}{c|c} \hline & \\ \hline & \end{array} \right].$$

OYOP: Essay: *Write on every second line, so that I can easily write between the lines.*

W2: Matrix $\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$, where \mathbf{A} and \mathbf{D} are 5×5 and 7×7 , resp. Suppose \mathbf{C} is the 7×5 zero-matrix. Prove that $\text{Det}(\mathbf{M}) = \text{Det}(\mathbf{A}) \cdot \text{Det}(\mathbf{D})$. [Hint: A good picture helps.]

Before starting your proof, state precisely the formula for determinant that you are using.

End of Class-W

W1: _____ 165pts

W2: _____ 65pts

Total: _____ 230pts

NAME:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: