Differential Eqns MAP2302 **W-Class** Prof. JLF King Wedn, 31Mar2021

Note. Write unambiguously e.g, $1/a + b$ should be *bracketed* either $[1/a] + b$ or $1/[a + b]$, as appropriate. (Be careful with **negative** signs!)

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

W1: Show no work.

a 10 The answer is:

Only one, but the light bulb has to want to change.

The question is: (See Canvas.)

How many psychiatrists does it take to change a light bulb?

b 20 $\lim_{x\nearrow\infty} \dfrac{\cosh(x+4)}{\sinh(x+7)} =$ ⌞. . . . . .⌟.

c 25 The *simplest* function $\varphi = \varphi(t)$ satisfying

†: $$\Big[\mathbf{D}^3 - 5\mathbf{I}\Big]^4(\varphi) \quad = \quad \mathsf{e}^{2t}$$

is $\varphi(t) =$ ⌞. . . . . .⌟.

[*Express your answer in simplest form.*]

d 15 "For *all* continuous functions h and g:
$[h+g]^{\circledast 2} = h^{\circledast 2} + 2[h \circledast g] + g^{\circledast 2}$." *T* *F*

e 30 With $G(t) := \cos\big(\cos(\cos(t))\big)$, *a* solution y=y(t) to $2y'' - 7y' + 3y = G$ is $y := \mathbf{f} \circledast G$, where $\mathbf{f}(t) =$ ⌞. . . . . .⌟.

f 10 30 Consider linear differential-operator

†: $$\mathsf{V}(y) := ty'' - [1+t]y' + y\,.$$

Verify [for yourself] that $\mathsf{V}(Y_0) = 0$ and $\mathsf{V}(Y_1) = 0$, where $Y_0 := \mathsf{e}^t$ and $Y_1 := 1 + t$. Their Wronskian (a fnc of t) is

$\mathcal{W}(Y_0, Y_1) =$ ⌞. . . . . .⌟.

Consider DE $\mathsf{V}(y) = 3t^2$. Dividing both sides by t makes the lefthand-side now have a monic operator, to which our VoP (Variation of Parameters) applies. Then VoP tells us that

$y_{\alpha,\beta} :=$ ⌞. . . . . .⌟

is the *general soln* to $\mathsf{V}(y_{\alpha,\beta}) = 3t^2$.

10 **W2:** *"I have neither requested nor received help on this exam other than from my professor."*

W1:	___ ___ ___	140pts
W2:	___ ___	10pts
Total:	___ ___ ___	150pts