

W4:	_____	60pts
W5:	_____	50pts
W6:	_____	45pts

**W4:** Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

**a** Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth.  one:

**True!** Yes! **wH'at S a?sEnTENcE**

**b** On a 5-element set, the number of reflexive symmetric binrels is .

On a 3-element set, there are  many equivalence relations.

**c** On  $\Omega := [1..29] \times [1..29]$ , define binary-relation **C** by:  $(x, \alpha) \mathbf{C} (y, \beta) \text{ IFF } x \cdot \beta \equiv_{30} y \cdot \alpha$ . Statement "Relation **C** is an equivalence relation" is:  **T**  **F**

OYOP: *In grammatical English sentences, write your essays on every third line (usually), so that I can easily write between the lines. Do not restate the question.* Start each essay on a new sheet-of-paper. Please number the pages "1 of 57", "2 of 57" ... (or "1/57", "2/57" ...) I suggest you put your name on each sheet.

**W5:** An **Lmino** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum  $N$ , let  $\mathbf{B}_N$  denote the  $3 \times N$  board: I.e,  is the  $\mathbf{B}_5$  board. Prove:

*Theorem: When  $N$  is odd, then board  $\mathbf{B}_N$  is not Lmino-tilable.*

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on  $N$  to prove the thm. Also: *Illustrate your proof* with (probably several) large, labeled pictures.

**W6:** Interval-of-integers  $\mathbf{J} := [101..200]$  has 99 elements. A subset  $S \subset \mathbf{J}$  is **Big** if  $|S| = 51$ . Subset  $S \subset \mathbf{J}$  is **Perfect** if there exist *distinct* members  $x, y \in S$  st.  $x + y = 300$ .

Prove that **Big  $\Rightarrow$  Perfect**. [Hint: PHP. Carefully specify what your pigeon-holes are.]

**Total:** \_\_\_\_\_ 155pts