Staple!

Team V____

Sets and Logic
MHF3202 8768

Home-V

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Due **BoC, Mon., 30Jan2017**, wATMP! Please *fill-in* every *blank* on this sheet. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

V1: *Show no work. Simply fill-in each blank on the problem-sheet.*

a Using *only* symbols $\boldsymbol{P}, \boldsymbol{Q}, \wedge, \vee, \neg, T, F,], [$, rewrite, in *simplest form*, expression $[[P \Rightarrow Q] \Rightarrow P]$ as ________. Ditto, rewrite $[P \Rightarrow [Q \Rightarrow P]]$ as ________.

b Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\ f(n) \subset \underbrace{\bigcup_{\ell=p-4}^{p+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

E3: ________. E2: ________. E1: ________.

c The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is ________.

[Write your answer as a product of powers and a multinomial. Optionally, you can expand the multinomial as a product of binomials.]

d $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. T F

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. T F

For all sets Ω, *there exists a fnc* $f\colon \mathbb{R} \to \Omega$. T F

For the three essay questions, carefully TYPE *triple–spaced, grammatical, solns. I suggest* LaTeX, *but other systems are ok too.*

V2: Kindergarten teacher Mrs. Nice has an unlimited supply of cookies. With her N pupils sitting in a circle, Mrs. Nice stands in front of *Abby*. Circling clockwise she takes one step, gives that student, *Bert*, a cookie. Continuing, she takes two steps (passing *Carol*), giving *Danny* a cookie. Then three steps, (passing *Englebert* and *Fran*), giving *Gail* a cookie. Four steps, cookie, five steps, cookie, and so on. [Some kids are getting lots of cookies.]

i Prove that if $N \geq 3$ is *prime*, then some child **never** gets a cookie. [*Hint:* PHP.]

ii What can you tell me: *When N is even? When N is a non-prime odd? When N has form 2^k? 3^k?* For what values of N does *Abby* get at least one cookie?

Let $V(N)$ be the number of kids who *neVer* get a cookie. How large can $V(N)$ get? [The above asked you to prove that $V(\text{OddPrime}) \geq 1$.] How large does ratio $\frac{V(N)}{N}$ get? What conjectures/theorems can you come up with? If you know how to program, do some computer experiments.

V3: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

$$\dagger\colon \qquad b_{n+2} := 9b_{n+1} - 18b_n\,, \quad \text{for } n = 0, 1, \ldots.$$

Use induction to prove, for each natnum k, that

$$\ddagger\colon \qquad b_k = 6^k - 3^k\,.$$

Further: Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 6 and 3 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $\boxed{b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n}$, for arbitrary real-number coefficients $\mathbf{S}$ and $\mathbf{P}$?

V4: Note $f(n) := \frac{1}{2} \cdot [5^n + 21^n]$ is an integer. Prove, for each *odd* $n \geq 3$, that $f(n)$ is composite. [*Hint:* Look at $f(n)$ mod something.]

[Are all the hypotheses necessary, or can some be weakened?]

End of Home-V

V1:	___ ___ ___	110pts
V2:	___ ___	80pts
V3:	___ ___	70pts
V4:	___ ___	65pts
Total:	___ ___ ___	325pts