Staple!

Sets and Logic MHF3202 2787	**Class-V**	Prof. JLF King *Touch*: 17Nov2018

V1:	___ ___	65pts
V2:	___ ___	85pts
V3:	___ ___	95pts
Total:	___ ___ ___	245pts

OYOP: For your 2 Essays: *Write your grammatical English sentences on every third line, so that I can easily write between the lines. Start each essay on a new sheet of paper.*

V1: Cantor Diagonalization Thm: *For each set B, there does not exist a surjection $B \twoheadrightarrow \mathcal{P}(B)$.*

V2: For $N \in \mathbb{Z}_+$, suppose $\Gamma = (V, E)$ is a complete digraph on N vertices. I.e, we have N towns, with each pair connected by a one-way road.

From a Γ-***good*** town $\mathbf{w} \in V$ we can legally bike to every town. A town $\mathbf{w}$ is Γ-***great*** if we can get to each town with a path of length $\leqslant 2$.

GD Prove that each such Γ has a Γ-good town.

GT Prove that each such Γ has a Γ-*great* town.

[*Hint:* Induction on N, works. In addition to text, your essay should have pictures illustrating your argument; they should be large. When refering to a good/great town, be careful to specify w.r.t. *what* network.]

V3: Short answer. Show no work.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a $\mathcal{P}(\mathcal{P}(\text{3-stooges}))$ has .... many elements.

b Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This $\approx$ is an equivalence-relation*" is: *T* *F*

c An explicit bijection $F: \mathbb{N} \hookrightarrow\!\!\!\!\!\twoheadrightarrow \mathbb{Z}$ is this:

When n is *even*, then $F(n) :=$ ....

When n is *odd*, then $F(n) :=$ ....

d To the interval $J := (-\frac{\pi}{2}, \frac{\pi}{2})$, define a bijection $g: (0,1) \hookrightarrow\!\!\!\!\!\twoheadrightarrow J$ by $g(x) :=$ ....

Using this g and a trigonometric fnc, define a bijection $h: (0,1) \hookrightarrow\!\!\!\!\!\twoheadrightarrow \mathbb{R}$ by $h(x) :=$ ....

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The map $f(k, n) := 2^k \cdot [1 + 2n]$ is a bijection from $\mathbb{N} \times \mathbb{N} \hookrightarrow\!\!\!\!\!\twoheadrightarrow \mathbb{Z}_+$. And $f^{-1}(112) = ($ .... , ... $)$.

End of Class-V