Staple!

Ord: ___ ___

Plex
MAA4402 8436

Class-V

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Notation. All sets are subsets of $\mathbb{C}$. For sets B and E, the difference set is $B \smallsetminus E := \{x \in B \mid x \notin E\}$.

The complement of E is $E^c := \mathbb{C} \smallsetminus E$.

For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

V1: Short answer. Show no work. $\mathbb{C}$-plane

a 10 10 Number $[\mathbf{i} + \sqrt{3}]^{70} = x + \mathbf{i}y$, for real numbers $x=$ ⌊.....⌋ and $y=$ ⌊.....⌋.

[*Multiplying complexes multiplies their moduli (absolute-values), and adds their angles.*]

b 15 10 Fnc $u(x,y) := \cos(y \cdot x) - 7x$ maps $\mathbb{R}\times\mathbb{R}\to\mathbb{R}$. Its Laplacian is $[\Delta(u)](x,y)=$ ⌊.....⌋.

There exists function $v{:}\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ such that $f(x+\mathbf{i}y) := u(x,y) + \mathbf{i}v(x,y)$ is holomorphic. *T F*

c 15 10 Write cos(-3**i**), which is real, ITOf exp() and *finite* add/sub/mul/div: cos(-3**i**)= ⌊.....⌋.

And cos(-3**i**) lies in circle the correct interval

$(-\infty, \frac{-1}{5}]$ $(\frac{-1}{5}, \frac{1}{5}]$ $(\frac{1}{5}, 2]$ $(2, 5]$ $(5, 15]$ $(15, 45]$ $(45, \infty)$

d 25 Compute the real $\alpha=$ ⌊.....⌋ such that

$$*: \quad 3^{\alpha} \cdot \sum_{k=0}^{1801} \binom{1801}{k} 2^k = \sum_{j=0}^{892} \binom{892}{j} 8^j .$$

[*Hint:* The Binomial Theorem]

e 1 9 10 The number of permutations of "PREPPER", as a multinomial coefficient, is ⌊.....⌋ ⌊.....⌋ numeral.

V2: Short answer. Metric space stuff

f 25

The empty-set is connected:	*T*	*F*
Punctured ball $\mathrm{PBal}_2(3\mathbf{i})$ *is connected:*	*T*	*F*
$\mathrm{Sph}_2(5\mathbf{i}) \cap \mathrm{Sph}_2(\mathbf{i})$ *is connected:*	*T*	*F*
$\mathrm{Sph}_2(4\mathbf{i}) \cup \mathrm{Sph}_2(-\mathbf{i})$ *is connected:*	*T*	*F*
$\mathrm{Sph}_2(5\mathbf{i}) \cup \mathrm{CldBal}_2(\mathbf{i})$ *is closed:*	*T*	*F*

g 25 All these sets are non-empty: Sets U and V open. Sets K, E and E_n are closed. Sets A and B are each path-connected.

$\exists q \in [A \cap B]$; *so* $A \cap B$ *is path-connected:*	*AT*	*AF*	*Nei*
Union $\bigcup_{n=1}^{\infty} E_n$ *is closed:*	*AT*	*AF*	*Nei*
Set $U \smallsetminus K$ *is open:*	*AT*	*AF*	*Nei*
Set $U \cup K$ *is open:*	*AT*	*AF*	*Nei*
Set $E \cap K$ *is closed:*	*AT*	*AF*	*Nei*

h 20 Cross-ratio $[z,\ 2{+}\mathbf{i},\ 4\mathbf{i},\ 3] = \frac{\mathtt{a}z + \mathtt{b}}{\mathtt{c}z + \mathtt{d}}$, where

a= ⌊.....⌋, b= ⌊.....⌋, c= ⌊.....⌋, d= ⌊.....⌋.

OYOP: *In grammatical English* ***sentences****, write your essay on every* 2^{nd} *line* (usually)*, so I can easily write between the lines.*

V3: For *reals* S, α, β, T, consider equation

$$\dagger: \quad S[x^2 + y^2] + \alpha x + \beta y + T = 0$$

in $\mathbb{R}\times\mathbb{R}$. Show that (†) describes a ***gen-circle*** [i.e, a *circle-or-line*; a ***generalized-circle***] IFF

$$*: \quad \alpha^2 + \beta^2 > 4ST .$$

End of Class-V

V1:	___ ___ ___	115pts
V2:	___ ___	70pts
V3:	___ ___	45pts
Total:	___ ___ ___	230pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*

Ord: ⌊.....⌋⌊.⌋⌊.⌋