

Read this. Use B^t for the transpose of B . When working over \mathbb{Z}_p , state answers using *symmetric residues*, e.g., in \mathbb{Z}_{13} , answers should lie in $[-6..6]$. Below, *eVal* means *eigenvalue*, *eVec* is *eigenvector* and *eSpace* is *eigenspace*.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: $\mathbf{DNE} \neq \{\} \neq 0$.

V1: Show no work.

10 **a** A multivariate polynomial, where each monomial has the same degree, is **Circle**: monogamous manic monic polyandrous delicious atrocious level flat uniform unitary Unitarian utilitarian **homogeneous** expialadocious penultimate smooth

10 **b** Perm $\pi := [7, 5, 2, 8, 6, 4, 1, 3]$ has $\text{Sgn}(\pi) = +1 -1$.

15 **c** Suppose C and A are 3×3 matrices s.t $\text{Det}(C) = \frac{1}{2}$ and $\text{Det}(A) = 5$. Then

$$\text{Det}(C^{-1}AC^t A^t AC^t) = \dots$$

30 **d** $M := \begin{bmatrix} 70 & 7 \\ 1 & 2 \end{bmatrix}$. Compute M^{-1} over these three fields.
[Write your \mathbb{Z}_p answers using *symmetric* residues.]

$$\text{Over } \mathbb{Z}_{13}: M^{-1} = \dots \text{. Over } \mathbb{Z}_7: M^{-1} = \dots$$

$$\text{Over } \mathbb{Q}: M^{-1} = \dots$$

15 10 15 15 **e** Here, matrices X, Y, P, Q range over $\text{MAT}_{2 \times 2}(\mathbb{R})$.

- 1** If 9 is an X^2 -eVal then 3 is an X -eVal. **T F**
- 2** If 3 is an X -eVal, then 9 is an X^2 -eVal. **T F**
- 3** If $X \sim Y$ and $P \sim Q$ then $XP \sim YQ$. **T F**
- 4** If $X \sim Y$ then $[X^2 + 5X] \sim [Y^2 + 5Y]$. **T F**

45 **f** Real $M := \begin{bmatrix} -5 & 3 & 18 \\ -2 & 0 & 12 \\ -4 & 4 & 7 \end{bmatrix}$ has characteristic polynomial

$$\varphi_M(t) = -t^3 + \dots t^2 + \dots t + \dots$$

Thus our M has three eigenvalues (all integers)

$$\alpha = \dots \leq \beta = \dots \leq \gamma = \dots \text{. Col-vec } \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ is a}$$

$$\gamma\text{-eVec for } M, \text{ where } x = \dots, y = \dots, z = \dots$$

OYOP: Essay: *Write on every second line, so that I can easily write between the lines.*

V2: **i** On your essay-paper, write “A 5×7 matrix M is in Reduced Row-Echelon Form IFF...” and complete the paragraph (with one or more sentences) to give a formal defn of RREF.

ii Give a careful proof of the...

1: RREF Uniqueness Theorem. Consider two 5×7 RREF matrices A and B . If A is row-equivalent to B , then $A = B$. \diamond

Start your argument with “Proof of the RREF Uniqueness Thm” and end it with “QED”.

End of Class-V

V1: _____ 165pts

V2: _____ 65pts

Total: _____ 230pts

NAME: _____

HONOR CODE: *I have neither requested nor received help on this exam other than from my professor.*

Signature: _____