

Linear Algebra Class-V Prof. JLF King  
MAS4105 14G8 Wednesday, 16Mar2022

**Read this.** Use  $B^t$  for the transpose of  $B$ . When working over  $\mathbb{Z}_p$ , state answers using *symmetric residues*, e.g, in  $\mathbb{Z}_{13}$ , answers should lie in  $[-6..6]$ . Below, **eVal** means *eigenvalue*, **eVec** is *eigenvector* and **eSpace** is *eigenspace*.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE**  $\neq \{\}$   $\neq 0$ .

**V1:** Show no work.

**a** A multivariate polynomial, where each monomial has the same degree, is Circle: **monogamous manic monic polyandrous delicious atrocious level flat uniform unitary Unitarian utilitarian**  
homogeneous expialadocious penultimate smooth

**b** Perm  $\pi := [7, 5, 2, 8, 6, 4, 1, 3]$  has  $\text{Sgn}(\pi) = +1 -1$ .

**c** Suppose  $C$  and  $A$  are  $3 \times 3$  matrices s.t  $\text{Det}(C) = \frac{1}{2}$  and  $\text{Det}(A) = 5$ . Then  
 $\text{Det}(C^{-1}AC^t A^t AC^t) =$  \_\_\_\_\_.

**d**  $M := \begin{bmatrix} 70 & 7 \\ 1 & 2 \end{bmatrix}$ . Compute  $M^{-1}$  over these three fields.  
[Write your  $\mathbb{Z}_p$  answers using *symmetric* residues.]

Over  $\mathbb{Z}_{13}$ :  $M^{-1} =$  \_\_\_\_\_. Over  $\mathbb{Z}_7$ :  $M^{-1} =$  \_\_\_\_\_.

Over  $\mathbb{Q}$ :  $M^{-1} =$  \_\_\_\_\_.

**e** Here, matrices  $X, Y, P, Q$  range over  $\text{MAT}_{2 \times 2}(\mathbb{R})$ .

- 1 If 9 is an  $X^2$ -eVal then 3 is an  $X$ -eVal.  $T$   $F$
- 2 If 3 is an  $X$ -eVal, then 9 is an  $X^2$ -eVal.  $T$   $F$
- 3 If  $X \stackrel{\text{sim}}{\sim} Y$  and  $P \stackrel{\text{sim}}{\sim} Q$  then  $XP \stackrel{\text{sim}}{\sim} YQ$ .  $T$   $F$
- 4 If  $X \stackrel{\text{sim}}{\sim} Y$  then  $[X^2 + 5X] \stackrel{\text{sim}}{\sim} [Y^2 + 5Y]$ .  $T$   $F$

**f** Real  $M := \begin{bmatrix} -5 & 3 & 18 \\ -2 & 0 & 12 \\ -4 & 4 & 7 \end{bmatrix}$  has characteristic polynomial

$\wp_M(t) = -t^3 +$  \_\_\_\_\_  $t^2 +$  \_\_\_\_\_  $t +$  \_\_\_\_\_.

Thus our  $M$  has three eigenvalues (all integers)

$\alpha =$  \_\_\_\_\_  $\leq \beta =$  \_\_\_\_\_  $\leq \gamma =$  \_\_\_\_\_. Col-vec  $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$  is a

$\gamma$ -eVec for  $M$ , where  $x =$  \_\_\_\_\_,  $y =$  \_\_\_\_\_,  $z =$  \_\_\_\_\_.

OYOP: Essay: *Write on every **second** line, so that I can easily write between the lines.*

**V2:** i On your essay-paper, write “A  $5 \times 7$  matrix  $M$  is in Reduced Row-Echelon Form *IFF* ...” and complete the paragraph (with one or more sentences) to give a formal defn of RREF.

ii Give a careful proof of the...

**1: RREF Uniqueness Theorem.** Consider two  $5 \times 7$  RREF matrices  $A$  and  $B$ . If  $A$  is row-equivalent to  $B$ , then  $A = B$ .  $\diamond$

Start your argument with “Proof of the RREF Uniqueness Thm” and end it with “QED”.

End of Class-V

**V1:** \_\_\_\_\_ 165pts

**V2:** \_\_\_\_\_ 65pts

**Total:** \_\_\_\_\_ 230pts

NAME: \_\_\_\_\_

**HONOR CODE:** “I have neither requested nor received help on this exam other than from my professor.”

Signature: \_\_\_\_\_