

Calc 3
MAC2313

Class-V

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V1: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a The arclength of curve $F(t) := (2t, t^2, \frac{1}{3}t^3)$ from $t=0$ to $t=3$ is $L := \int_0^3 G(t) dt$, where $G(t)=$ _____.
Computing, $L =$ _____.

b The radius-of-curvature of the exponential curve $x \mapsto e^{Mx}$ at $(0, 1)$ is RoC= _____.

c The dark planet MALFOY has two satellites in circular orbit, CRABBE and GOYLE. CRABBE's orbital-speed is 1000mph and his orbital-radius is 27000mi. GOYLE's orb-speed is 1500mph, so his orb-radius is _____ mi.
Out in CRABBE's orbit, the from-MALFOY escape-speed is _____ mph.
(You may express this answer as an arithmetic combination of 1000, 27000, 1500 and other numbers.)

d The plane through points $\mathbf{p} := (2, 1, 3)$, $\mathbf{q} := (1, 2, 3)$ and $\mathbf{r} := (3, 0, 1)$ can be written in form $Ax + By + Cz = V$, for numbers $A=$ _____, $B=$ _____, $C=$ _____, and $V=$ _____. The point on this plane which is closest to the origin is (_____, _____, _____).

e A speed S parameterization of the radius R circle in the plane is $\mathbf{H}(t) :=$ _____.

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Do not restate the question.

V2: For a smooth fnc $\mathbf{F}: \mathbb{R} \rightarrow \mathbb{R}^3$, interpreted as mapping "time" to "position", define velocity $\mathbf{V}(t) := \mathbf{F}'(t)$ and

$$\mathbf{T}(t) := \frac{\mathbf{V}(t)}{\|\mathbf{V}(t)\|} \quad \text{and} \quad \mathbf{A}(t) := \mathbf{V}'(t).$$

We defined the curvature as $\kappa(t) := \|\mathbf{T}'(t)\|/\|\mathbf{V}(t)\|$.

In English sentences structured into beautiful paragraphs, derive that the curvature function also equals

$$*: \quad \kappa = \frac{\|\mathbf{V} \times \mathbf{A}\|}{\|\mathbf{V}\|^3}.$$

Likely, you will want to carefully *state*, then *prove*, an intermediate result, before establishing (*).

End of Class-V

V1: _____ 180pts

V2: _____ 85pts

Total: _____ 265pts

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor (or his colleague)."
Name/Signature/Ord

Ord: _____