Staple!

Sets and Logic MHF3202 2787	**Class-U**	Prof. JLF King 6Sep2015

OYOP: *Write each essay on new sheets of paper, writing every **third** line, so that I can easily write between the lines. In grammatical English **sentences**, prove the following:*

U1: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

$$\dagger: \quad b_{n+2} := 7b_{n+1} - 10b_n, \quad \text{for } n = 0, 1, \ldots.$$

Use induction to prove, for all $k \in [0\,..\,\infty)$, that

$$\ddagger: \quad b_k = 5^k - 2^k.$$

Further. Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $\boxed{b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n}$, for arbitrary real-number coefficients **S** and **P**?

U2: For a posint K, let $\equiv$ mean $\equiv_K$. DEFN: Expression "$x \equiv y$" means. . . .

Please prove: THM: *For all* $b, \beta, g, \gamma \in \mathbb{Z}$, *if* $b \equiv \beta$ *and* $g \equiv \gamma$ *then* $[b \cdot g] \equiv [\beta \cdot \gamma]$.

U3: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Mod K:=229, the recipr. $\langle \frac{1}{45} \rangle_K$= .... $\in[0\,..\,K)$.

[*Hint:* ⸸] So x= .... $\in[0\,..\,K)$ solves $4 - 45x \equiv_K 7$.

b Posints K= ....., N= ....., α= ....., β= .....,

are st. $\alpha \equiv_K \beta$, yet N^α= ..... is **not** $\equiv_K$ to N^β= ......

c Stmt $C \Rightarrow B$ has *contrapositive* .... and *converse* .... Recall &, ∨, ¬ mean AND, OR, NOT.

Using *only* symbols $\wedge, \vee, \neg, \boldsymbol{B}, \boldsymbol{C},], [$,

write $C \Rightarrow B$ as ....

d Define G:[1 .. 12]↺ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is .... The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ....

e $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. *T* *F*

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. *T* *F*

For all sets Ω, *there exists a fnc* $f : \mathbb{R} \to \Omega$. *T* *F*

End of Class-U

U1:	___ ___	60pts
U2:	___ ___	30pts
U3:	___ ___ ___	150pts
Total:	___ ___ ___	240pts

An extra question, for your posting pleasure:

f Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}:\ f(n) \subset \underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: .... $E2$: .... $E3$: ....