Linear Algebra
MAS4105 *14G8* **Class-U** Prof. JLF King
Wednesday, 16Feb2022

Hello. Use L_M for the lefthand-action of matrix M. Use B^t for the transpose of B. When working over $\mathbb{Z}_p$, state answers using *symmetric residues*, e.g, in $\mathbb{Z}_{13}$, answers should lie in $[-6 .. 6]$.

Write **DNE** <u>if</u> the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

U1: Show no work.

y **5** Prof. King wears bifocals, and cannot read small handwriting. Circle one: **True!** **Yes!** ***Who??***

z **10** Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, $500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is mid-March, 2023.

Circle : **Yes** **True** ***Résumé material!***

a **25** Shear the plane *vertically*, sending $\mathbf{e}_1$ to $\mathbf{e}_1 + 3\mathbf{e}_2$, followed by the *horizontal* shear which sends $\mathbf{e}_2$ to $-2\mathbf{e}_1 + \mathbf{e}_2$. Let S be the 2×2 matrix whose lefthand action is the preceding composition of shears.

Then $S = \left[\rule{0pt}{40pt} \quad\quad\quad\quad\quad \right]$.

b **30** Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) := \begin{bmatrix} 3x - y \\ 2x + 6y \end{bmatrix}$. W.r.t ordered-basis $\mathcal{B} := \left(\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right)$, let $M := [\![T]\!]_{\mathcal{B}}^{\mathcal{B}}$. Then $M = RTR^{-1}$,

where $R = \left[\rule{0pt}{30pt} \quad\quad\quad \right]$, $M = \left[\rule{0pt}{30pt} \quad\quad\quad \right]$.

c **25** In each blank below, write either "there exist" or "for all", Circle one of the underlined scalar-pairs, and Circle a phrase.

Assertion $\boxed{\mathrm{Spn}(\mathbf{v}, \mathbf{w}) \supset \mathrm{Spn}(\mathbf{x}, \mathbf{y})}$ *means:*

" .... scalars $\underline{a, b} \mid \underline{c, d}$ (st. | we have that | and)

.... scalars $\underline{a, b} \mid \underline{c, d}$ (st. | we have that)

$\boxed{a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}}$."

d **20** The 3×3 elem-matrix whose lefthand action adds 8 times row-2 to row-1 is $\left[\rule{0pt}{50pt} \quad\quad\quad\quad \right]$.

U2: OYOP: Essay: *Write on every **second** line, so that I can easily write between the lines.*

i Consider a lin.trn $T:\mathbf{X} \to \mathbf{U}$ between finite-dimensional VSes. Distinguishing between zero-vectors $\vec{\mathbf{0}}_{\mathbf{X}}$ and $\vec{\mathbf{0}}_{\mathbf{U}}$, give, using set-builder notaion: *A formal defn of* $\mathrm{Range}(T)$. And: *A formal defn of* $\mathrm{Nul}(T)$.

ii State the Rank+Nullity theorem for $T:\mathbf{X} \to \mathbf{U}$.

iii Give a careful proof of the Rank+Nullity thm for $T:\mathbf{X} \to \mathbf{U}$. Also: Use good, large *pictures* to illustrate the ideas in the proof.

End of Class-U

U1: ___ ___ ___ 115pts

U2: ___ ___ 75pts

Total: ___ ___ ___ 190pts

NAME:
....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:
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