Differential Eqns MAP2302 **T-Class** Prof. JLF King Wedn., 17Feb2021

T1: ___ ___ ___ 150pts

Total: ___ ___ ___ 150pts

T1: Show no work.

15 15 a Number $\left[\mathbf{i}+\sqrt{3}\right]^{70} = x+\mathbf{i}y$, for real numbers $x=$ ⌞.....⌟ and $y=$ ⌞.....⌟.

[*Multiplying complexes multiplies their moduli (absolute-values), and adds their angles.*]

10 10 10 0 b *A* particular polynomial $p{=}p(t)$ satisfying

$$*: \qquad p' + 2p = 6t^2+8t+1$$

is $p(t) =$ ⌞.....⌟ $\cdot t^2 +$ ⌞.....⌟ $\cdot t +$ ⌞.....⌟.

The general soln has form $y_\alpha(t) = \alpha \mathsf{e}^{Mt} + p(t)$, where $M=$ ⌞.....⌟. [Put correct numbers in the four blanks.]

10 10 10 c **A** soln (use PolyExp) to $y''+y = 2t^2\mathsf{e}^{-t}$ is $y(t)=$ ⌞.....⌟.

6 6 6 6 d Operators $\mathsf{V}, \mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ map from $\mathbf{C}^\infty \to \mathbf{C}^\infty$, and V is linear. The other maps are

$$\mathbf{P}(f) \coloneqq [t \mapsto f(t)+3], \quad \mathbf{Q}(f) \coloneqq [t \mapsto f(t+3)],$$
$$\mathbf{R}(f) \coloneqq [t \mapsto f\big(f(t)\big)], \quad \mathbf{S}(f) \coloneqq \mathsf{V}\big(\mathsf{V}(f)\big),$$

Then ... **P** *is linear:* *T F*. **Q** *is linear:* *T F*.
R *is linear:* *T F*. **S** *is linear:* *T F*.

5 5 5 5 5 5 e Let $U \coloneqq 3-2\mathbf{i}$ and $W \coloneqq 4+\mathbf{i}$. The gen.soln to a CCLDE is $\boxed{y_{\alpha,\beta}(t) = \alpha\cdot\mathsf{e}^{Ut} + \beta\cdot\mathsf{e}^{Wt}}$. The CCLDE that every such $y()$ satisfies is

⌞.....⌟ $= 0$.

[Fill-in the blank with the appropriate sum of derivatives-of-y times various numbers, which may be complex.]

6 f *"I have neither requested nor received help on this exam other than from my professor."*