



Staple!

Ord: _____

Differential Eqns **T-Class** Prof. JLF King
MAP2302 Wed.27Feb2019

Welcome. Write **DNE** if the object does not exist or the operation cannot be performed. NB: $\mathbf{DNE} \neq \{\} \neq 0$. Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g., write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$.

T1: Show no work.

a The Logistic model is credited to Circle: Argand Abel Cauchy Eigen Euler Gauss Heaviside Hypatia Lagrange Laplace Lindelöf Logos Malthus Maclaurin Picard Taylor Verhulst Weierstrass Zeno

b Complex number $[x + iy]^2 = -32i$, for real numbers $x > y$, where $x = \underline{\hspace{2cm}}$ and $y = \underline{\hspace{2cm}}$.

c ITOf $\sqrt{\cdot}$, $\exp()$, $\log()$, add/sub/mul/div, write, in simplest form: $\operatorname{acosh}(\frac{5}{3}) = \underline{\hspace{2cm}}$ and $\operatorname{asinh}(\frac{5}{3}) = \underline{\hspace{2cm}}$.

d U.F. $y = y(x)$ satisfies $y(\pi) = 1$ and

$$[2y + x \cos(xy)]y' + y \cos(xy) = 2.$$

It satisfies $\mathbf{F}(x, y(x)) = \beta$ for number $\beta = \underline{\hspace{2cm}}$ and

function $\mathbf{F}(x, y) := \underline{\hspace{2cm}}$.

e A hanging cable has vertex-tension $T = 200 \text{ lb}$ and cable-density 2 lb/ft . ITOf exp, log add/sub/mul/div, [write in simplest form] the length of cable lying above $[0\text{ft}, 800\text{ft}]$ is $\underline{\hspace{2cm}}$ ft. [Vertex=0ft]

f

A *critically-damped* unforced spring has DE

*: $My'' + \mathbf{B}y' + \mathbf{K}y = 0 \frac{\text{kg}\cdot\text{m}}{\text{sec}^2}$, where $\mathbf{M} := 3\text{kg}$, and the Hooke's constant is $\mathbf{K} := 75 \frac{\text{kg}}{\text{sec}^2}$.

The damping constant $\mathbf{B} = \underline{\hspace{2cm}}$.

The *general soln* to critically-damped (*) is

$$y(t) = \left[\alpha \cdot \underline{\hspace{2cm}} + \beta \cdot \underline{\hspace{2cm}} \right] \text{m.}$$

Here, $\alpha, \beta \in \mathbb{R}$, dimensionless. [The above blanks have numbers & units in various places; the bracketed quantity is dimensionless. Is $\exp(\cdot)$ more convenient than e^{\cdot} notation?] The **specific** soln with $y(0\text{sec}) = 0\text{m}$ and $y'(0\text{sec}) = 2 \frac{\text{m}}{\text{sec}}$ has

$$\alpha = \underline{\hspace{2cm}}, \beta = \underline{\hspace{2cm}}.$$

OYOP: In grammatical English **Sentences**, write your essay on every 2nd line (usually), so I can easily write between the lines.

T2: Fnc $h = h(x)$ describes the height of a hanging cable, where $x \text{:: ft}$ and $h(x) \text{:: ft}$, and the cable's vertex is at $(0\text{ft}, 0\text{ft})$. Using $\mathbf{T} \text{:: lb}$ for the *vertex-tension*, and $\mathbf{S} \text{:: } \frac{\text{lb}}{\text{ft}}$ for the *cable-density*, write an essay in text and LARGE, LABELED pictures, carefully deriving our DE for $h()$. [Do not solve the DE.]

End of T-Class

T1: _____ 175pts

T2: _____ 65pts

Total: _____ 240pts

Please PRINT your **name** and **ordinal**. Ta:

Ord: _____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: _____