Differential Eqns MAP2302 3228	**T-Class**	Prof. JLF King Wedn., 17Oct2018

T1: *Show no work.*

a Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, $500 total] is a *really good idea*, and the due date for the emailed-PDF is typically mid-March. Circle:

Yes **True** ***Résumé material!***

b Bacteria with birth-multiplier $\mathbf{B} :: \frac{1}{\mathsf{min}}$ are in a petri dish with carrying capacity $\mathbf{C} :: \mathsf{oz}$. The population, $p(t) :: \mathsf{oz}$, satisfies the Logistic DE.

The DE is ....

For *Hysteria* bacteria, $\mathbf{B} = \frac{1/5}{\mathsf{min}}$. This petri dish has $\mathbf{C}$=16oz, with initial population $\mathbf{p}_0 = 2\mathsf{oz}$. The time when *Hysteria* has reached half the carrying capacity

is .... min $\approx$ ...... min.

[NB: You may use exp() and log() to express your answer.]

C Fnc $y_{\alpha,\beta}(t) = \alpha e^{At} + \beta e^{Bt} + P \cdot \sin(t) + Q \cdot \cos(t)$
is the general soln to

$$*: \qquad 3y'' + 4y' + y = \cos(t), \quad \textit{with numbers}$$

$A =$ ______________, $B =$ ______________, $P =$ _____, $Q =$ _____.

Also, the *constants* on LhS($*$) are $3, 4, 1$. With the DE describing the position of a spring, the *constant* corresponding to Hooke's constant is ________.

d Operators $\mathsf{V}, \mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ map from $\mathbf{C}^\infty \to \mathbf{C}^\infty$, and V is linear. The other maps are

$$\mathbf{P}(f) := [t \mapsto f(t)+3], \quad \mathbf{Q}(f) := [t \mapsto f(t+3)],$$
$$\mathbf{R}(f) := [t \mapsto f\big(f(t)\big)], \quad \mathbf{S}(f) := \mathsf{V}\big(\mathsf{V}(f)\big),$$

Then... **P** *is linear:* *T F*. **Q** *is linear:* *T F*.
R *is linear:* *T F*. **S** *is linear:* *T F*.

e With $v := \exp(-2+5\boldsymbol{i})$, then $|v| =$ ______________ .

This $|v|$ lies in [circle the correct interval]

$$\big[0, \tfrac{1}{2}\big), \quad \big[\tfrac{1}{2}, 1\big), \quad \big[1, 2\big), \quad \big[2, 4\big), \quad \big[4, 8\big), \quad \big[8, \infty\big).$$

T2: *Show no work.*

i A tank initially has 80gal of salinity $2\frac{\text{lb}}{\text{gal}}$ brine. Pipe-1 feeds the tank, at rate $3\frac{\text{gal}}{\text{min}}$, with salinity $1\frac{\text{lb}}{\text{gal}}$ brine. Pipe-2 feeds at $2\frac{\text{gal}}{\text{min}}$ with salinity $2\,\frac{\text{lb}}{\text{gal}}$. The tank discharges brine at $9\frac{\text{gal}}{\text{min}}$. Until the tank empties, it holds $W(t)=$ [.................] gal; it empties in .... min.

The amount, $y(t)$, of lb of salt in the tank at time t, satisfies FOLDE $\frac{dy}{dt} + C(t)\cdot y = G(t)$, where $C(t)=$ .... and $G(t)=$ ....

ii A *critically-damped* unforced spring has DE

$*$: $\mathbf{M}y'' + \mathbf{B}y' + \mathbf{K}y = 0\,\frac{\mathsf{kg\cdot m}}{\mathsf{sec}^2}$, *where* $\mathbf{M} := 3\mathsf{kg}$, *and the Hooke's constant is* $\mathbf{K} := 75\frac{\mathsf{kg}}{\mathsf{sec}^2}$.

The damping constant $\mathbf{B}$= ______________________.

The *general soln* to critically-damped $(*)$ is

$$y(t) = \Big[\alpha\cdot \_\_\_\_\_\_\_\_\_\_ + \beta\cdot \_\_\_\_\_\_\_\_\_\_\Big]\,\mathsf{m}.$$

Here, $\alpha,\beta \in \mathbb{R}$, dimensionless. (*The above blanks above have units & numbers in various places; the bracketed quantity is dimensionless.* Is exp(?) is more convenient than $\mathsf{e}^?$ notation?) The **specific** soln with $y(0\mathsf{sec}) = 0\mathsf{m}$ and $y'(0\mathsf{sec}) = 2\,\frac{\mathsf{m}}{\mathsf{sec}}$ has

α= ______________________, β= ______________________.

T1:	___ ___ ___	115pts
T2:	___ ___	70pts
Total:	___ ___ ___	185pts