Ord: __ __

Differential Eqns MAP2302 — **T-Class** — Prof. JLF King, Wedn, 28Feb2018

Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with negative signs!)**

T1: Show no work.

a A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is two courses. Circle:

Yes **True** **Darn tootin'!**

b Fnc $y_{\alpha,\beta}(t) = \alpha e^{At} + \beta e^{Bt} + P\cdot\sin(t) + Q\cdot\cos(t)$ is the general soln to

$$*: \quad 3y'' + 5y' + y = \cos(t), \quad \textit{with numbers}$$

$A=$ ____ , $B=$ ____ , $P=$ ____ , $Q=$ ____ .

Also, the *constants* on LhS(∗) are 3, 5, 1. With the DE describing the position of a spring, the *constant* corresponding to Hooke's constant is ____ .

c DE $\left[[2x+8]y\cdot\frac{dy}{dx}\right] + 4y^2 = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc

$V(y) =$ ____

gives a *new* DE which *is* exact.

Solving the exact-DE, every soln $y=y(x)$ satisfies $F\big(x, y(x)\big) = \alpha$, for some constant α, where

$F(x,y) =$ ____ .

d Let $U := 3-2\boldsymbol{i}$ and $W := 4+\boldsymbol{i}$. The gen.soln to a CCLDE is $y_{\alpha,\beta}(t) = \alpha\cdot e^{Ut} + \beta\cdot e^{Wt}$. The CCLDE that every such $y()$ satisfies is

____ $= 0$.

[*Hint:* Fill-in the blank with the appropriate sum of derivatives-of-y times various constants.]

e A *critically-damped* unforced spring has DE

$$*: \quad \mathbf{M}y'' + \mathbf{B}y' + \mathbf{K}y = 0\,\frac{\text{kg}\cdot\text{m}}{\text{sec}^2}, \quad \textit{where}$$
$$\mathbf{M} := 3\text{kg}, \textit{ and the Hooke's constant is } \mathbf{K} := 75\frac{\text{kg}}{\text{sec}^2}.$$

The damping constant $\mathbf{B}=$ ____ .

The *general soln* to critically-damped (∗) is

$y(t) = \big[\alpha\cdot$ ____ $+ \beta\cdot$ ____ $\big]$m.

Here, $\alpha,\beta \in \mathbb{C}$. (*The 3 blanks will have units & numbers in various places.* Maybe exp(?) is more convenient than $e^?$ notation.) The **specific** soln with $y(0\text{sec}) = 0$m and $y'(0\text{sec}) = 2\,\frac{\text{m}}{\text{sec}}$ has $\alpha=$ ____ , $\beta=$ ____ .

T2: Show no work.

f A tank initially holds 80gal of $2\frac{\text{lb}}{\text{gal}}$ brine. Pipe-1 feeds the tank, at rate $3\frac{\text{gal}}{\text{min}}$, with brine of time-varying salinity $\left[\frac{t}{\text{min}}\right]^3\frac{\text{lb}}{\text{gal}}$. Pipe-2 feeds the tank at $2\frac{\text{gal}}{\text{min}}$, brine of salinity $e^{t/\text{min}}\frac{\text{lb}}{\text{gal}}$. The tank discharges brine at rate $9\frac{\text{gal}}{\text{min}}$. Until the tank empties, the tank holds $W(t) = \big[$ ____ $\big]$gal; it empties in ____ min.

Finally, $y(t)$, the number of pounds of salt in the tank at time t, satisfies FOLDE $\frac{dy}{dt} + F(t)\cdot y = H(t)$,

where $F(t)=$ ____

and $H(t)=$ ____ .

T1:	___ ___ ___	130pts
T2:	___ ___	40pts
Total:	___ ___ ___	170pts