Differential Eqns MAP2302 **T-Class** Prof. JLF King Wedn, 18Oct2017

Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

T1: Show no work.

a Which is *optional*? (circle) *Writing-in-sentences.* *Writing-t-different-from-+.* *Writing-LARGE.* *Um...*

b *Hysteria* bacteria, with birth-multiplier $\mathbf{B} :: \frac{1}{\mathsf{min}}$, are in a petri dish with carrying capacity $\mathbf{C} :: \mathsf{oz}$. The population, $p(t) :: \mathsf{oz}$, satisfies the Logistic DE.

The DE is _____.

For *Hysteria*, $\mathbf{B} = \frac{1/5}{\mathsf{min}}$. This petri dish has $\mathbf{C}$=16oz, with initial population $\mathbf{p}_0 = 2\mathsf{oz}$. The time when *Hysteria* has reached half the carrying capacity

is _____ min $\approx$ _____ min.

[NB: You may use exp() and log() to express your answer.]

c The simplest soln to $y'' + 2y' + y = [t^2 + 1]/\mathsf{e}^t$ is $y(t) =$ _____.

d $\left[x^2 \circledast \mathsf{e}^{3x}\right] =$ _____.

e DiffOperators $\mathbf{P}, \mathbf{Q}, \mathbf{R}, \mathbf{S}$ are defined as

$$\mathbf{P}(f) := f(3) \cdot f', \qquad \mathbf{Q}(f) := \cos(3) \cdot f^{(3)},$$
$$\mathbf{R}(f) := [\cos(3) \cdot f] + f'', \qquad \mathbf{S}(f) := \cos(3) + [3f'].$$

Then... **P** *is linear:* T F. **Q** *is linear:* T F.
R *is linear:* T F. **S** *is linear:* T F.

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

T2: Brine with salinity $\mathsf{S} := 5\frac{\mathsf{lb}}{\mathsf{gal}}$ flows at $\mathsf{R} := 2\frac{\mathsf{gal}}{\mathsf{min}}$ into Tank-1, which discharges at rate R into Tank-2, which discharges at R. Each contains $\mathsf{U} := 50\mathsf{gal}$ of brine; initial-salinity $1\frac{\mathsf{lb}}{\mathsf{gal}}$ for Tank-1, and initial-salinity $2\frac{\mathsf{lb}}{\mathsf{gal}}$ for Tank-2.

OYOP, draw a large, **carefully labelled** picture of the ocean+tanks.

Derive the DE for $\sigma_1(t)$, the salinity in Tank-1; it is _____.

ITOf $\sigma_1(t)$, derive the DE for $\sigma_2(t)$, the salinity in Tank-2:

It is _____.

Solve the first DE, getting

$\sigma_1(t) =$ _____.

End of T-Class

T1: ___ ___ ___ 110pts

T2: ___ ___ 85pts

Total: ___ ___ ___ 195pts