# Solutions to selected problems in MAP 6472

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## 1 Problem 8

We know that $\omega \in \bigcap_{\beta \in B} \bigcup_{\alpha \in A} S_{\beta,\alpha}$ if and only if for each $\beta \in B$ there exists an $\alpha \in A$ such that $\omega \in S_{\beta,\alpha}$.

Let $\omega \in \bigcup_{f \in A^B} \bigcap_{\beta \in B} S_{\beta,f(\beta)}$. Then there exists a function $g : B \to A$ such that for all $\beta \in B$, $\omega \in S_{\beta,g(\beta)}$. Since $g(\beta) \in A$, we get from the first statement that $\omega \in \bigcap_{\beta \in B} \bigcup_{\alpha \in A} S_{\beta,\alpha}$. Thus $\bigcup_{f \in A^B} \bigcap_{\beta \in B} S_{\beta,f(\beta)} \subset \bigcap_{\beta \in B} \bigcup_{\alpha \in A} S_{\beta,\alpha}$.

Let $\omega \in \bigcap_{\beta \in B} \bigcup_{\alpha \in A} S_{\beta,\alpha}$. Then for each $\beta \in B$, we have a non-empty subset of $A_\beta$ of $A$ such that $\omega \in S_{\beta,\alpha}$ for all $\alpha \in A_\beta$. By the axiom of choice, there exists a function $f : B \to A$, which is equal to exactly one element of $A_\beta$ for each $\beta$. Thus $\omega \in S_{\beta,f(\beta)}$ for all $\beta \in B$. This implies that $\omega \in \bigcap_{\beta \in B} S_{\beta,f(\beta)}$ and furthermore $\omega \in \bigcup_{f \in A^B} \bigcap_{\beta \in B} S_{\beta,f(\beta)}$. Thus $\bigcap_{\beta \in B} \bigcup_{\alpha \in A} S_{\beta,\alpha} \subset \bigcup_{f \in A^B} \bigcap_{\beta \in B} S_{\beta,f(\beta)}$.

Hence $\bigcap_{\beta \in B} \bigcup_{\alpha \in A} S_{\beta,\alpha} = \bigcup_{f \in A^B} \bigcap_{\beta \in B} S_{\beta,f(\beta)}$.