Mindbenders posed in MAP 6472

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1. **Who's the Toss?** Alan and Barb each have a fair coin. Alan tosses his coin 2000 times while Barb tosses hers 2001 times. Barb wins only if the number of heads in her sequence of tosses is *strictly* greater than the number of heads in Alan's sequence. What is the probability Barb wins?

2. **Winners never quit, quitters never win.** Prof. J and Alla are playing a game of chance. Prof. J begins the game by throwing a \$1 bill on a table and asking Alla, "Wanna quit now?". If Alla decides to quit, she keeps the dollar bill and the game ends. If she decides not to, Prof. J flips a fair coin. If the outcome is Tails, he takes away the money and the game ends. If the outcome is Heads, he triples the money on the table and once again asks Alla, "Wanna quit now?". If Alla decides to quit she keeps the money and if not Prof. J flips a coin and the game continues. Now suppose that Alla has a coin of her own which when flipped shows Heads with probability p and Tails with probability $q = 1 - p$. Also suppose that every time Alla is asked the question by Prof. J, she secretly flips her coin and decides to quit if a Heads shows up and decides to continue if a Tails appears. Find all $q \in [0, 1]$ such that the expected amount of money which Alla wins is infinite.

3. **The Uneducated Soldiers Problem** Suppose there are N uneducated soldiers standing in a row. Each of them flips a fair coin and turns right if a *Heads* appears and turns left if a *Tails* appears. Now if a soldier finds himself facing another soldier, he turns around. If he is facing another soldier's back or empty space (for the case of the soldiers at the endpoints), he stays put. Further suppose that the soldiers do the "turning around" or "staying put" at the same time after every second.

   (a) Prove that the situation always stabilizes.

   (b) Find the expected waiting time for stabilization.

   (c) Show that the final state after stabilization always has the form : *L soldiers facing the left side followed by R soldiers facing the right side*, where R+L = N. What is the probability distribution of R?

4. Suppose that a drunk D is at the origin on the real line at time $t = 0$. After every second, he tosses a fair coin and takes a unit step to the right if *Heads* appears and takes a unit step to the left if *Tails* appears. Suppose that there are two walls at positions $-l$ and r on the real line where l, r are non-negative integers. Find $H(l, r)$ where is the $H(l, r)$ is

the expected time of the event that D hits a wall. Note that $H(0,0) = 0$, $H(0,r) = 0$ and $H(1,1) = 1$. *Hint*: Get a recurrence relation.