SeLo
MHF3202 139A **Quizzes P** Friday 26Aug2022

Generalized parentheses. **Common:**

( ) *Parentheses.* [ ] *Brackets.* ⟨ ⟩ *Angle-brackets.* { } *Braces.* | | *Vertical bars* (*abs.value, cardinality*).

Less common: ⌊ ⌋ *Floor.* ⌈ ⌉ *Ceiling.* ‖ ‖ *Norm-of.* ⟦ ⟧ *Double-bracket.* ⟪ ⟫ *Double-angle-bracket.*

Number Sets.

Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers $= \{0, 1, 2, \ldots\}$.

$\mathbb{Z}$ = integers $= \{\ldots, -2, -1, 0, 1, \ldots\}$. For the set $\{1, 2, 3, \ldots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers $= \{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** *and* $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$. And $[-\infty \,..\, -1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathrm{i}| = \sqrt{29}$.

Mathematical objects.

Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2] \cdot \mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2] \cdot \mathrm{e}^t$ is a polyexp-sum.

Phrases.

WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※ ="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Sequence notation.

A *sequence* $\vec{\mathbf{x}}$ abbreviates $(x_0, x_1, x_2, x_3, \ldots)$. For a set Ω, expression "$\vec{\mathbf{x}} \subset \Omega$" means $[\forall n \colon x_n \in \Omega]$. Use $\mathrm{Tail}_N(\vec{\mathbf{x}})$ for the subsequence

$$(x_N, x_{N+1}, x_{N+2}, \ldots)$$

of $\vec{\mathbf{x}}$. Given a fnc $f \colon \Omega \to \Lambda$ and an Ω-sequence $\vec{\mathbf{x}}$, let $f(\vec{\mathbf{x}})$ be the Λ-sequence $(f(x_1), f(x_2), f(x_2), \ldots)$.

Suppose Ω has an addition and multiplication. For Ω-seqs $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$, then, let $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{\mathbf{x}} + \vec{\mathbf{y}} = [n \mapsto [x_n + y_n]] .$$

Similarly, $\vec{\mathbf{x}} \cdot \vec{\mathbf{y}}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

SeLo [2022t] quizzes

(Recall, the lowest MQ score is dropped. Consequently, there is no make-up for the first missed MQ.)

P1: Fri. 02 Sep $\mathcal{P}(\mathcal{P}(\text{3-stooges}))$ has ________ elements.

P2: Fri. 09 Sep Binomial coefficient $\binom{7}{4}$ = ________ = ________.

The number of permutations of "PREPPER", as a multinomial coefficient, is ________ $\overset{\text{numeral}}{\equiv}$ ________.

P3: Wed. 14 Sep The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is ________.

[Write your answer as a product of powers and a multinomial. Optionally, you can expand the multinomial as a product of binomials.]

P4: Wed. 14 Sep

The number of ways of picking 4 objects from 8 types is $\left[\!\left[{8 \atop 4}\right]\!\right] \overset{\text{Binom}}{\underset{\text{coeff}}{\equiv}} \big($ ________ $\big) \overset{\text{Integer}}{\underset{\text{numeral}}{\equiv}}$ ________.

And $\left[\!\left[{8 \atop 4}\right]\!\right] = \left[\!\left[{T \atop K}\right]\!\right]$, where T= ________ $\neq 8$, and K= ________ $\neq 4,1$.

P5: Mon. 17 Oct *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

"I wouldn't miss it for the world!"

P6: Mod. 31 Oct For a finite list $\mathcal{S}$ of posints, define

$$\boldsymbol{\mu}_{\mathcal{S}}(N) := \left\{ k \in [1..N] \;\middle|\; \exists d \in \mathcal{S} \text{ with } d \bullet\!| \, k \right\}.$$

Use Inclusion-Exclusion to compute

$\left|\mu_{\{3,5,7\}}(96)\right|$ = ________.

P7: Fri. 18 Nov An explicit bijection $\psi{:}\mathbb{Z}\hookrightarrow\!\!\!\!\!\twoheadrightarrow\mathbb{N}$ is this:

If $n \geq 0$, then $\psi(n) :=$ ________.

If $n < 0$, then $\psi(n) :=$ ________.

P8: Mon. 21 Nov Let $\mathbb{S}_{\mathbb{N}}$ be the set of *permutations of* $\mathbb{N}$. Circle those of following sets which are equinumerous with $\mathbb{N}^{\mathbb{N}}$:

$\mathbb{N}$ $\mathbb{R}$ $\mathbb{N}\times\mathbb{R}$ $2^{\mathbb{R}}$ $\mathbb{R}^{\mathbb{N}}$ $\mathbb{R}^{\mathbb{R}}$ $\mathbb{S}_{\mathbb{N}}$

[Schröder-Bernstein is useful for some of these.]

P9: Mon. 28 Nov To the interval $J := (-\frac{\pi}{2}, \frac{\pi}{2})$, define a bijection $g\colon (0,1) \hookrightarrow\!\!\!\!\rightarrow J$ by $g(x) :=$ ....

Using this g and a trigonometric fnc, define a bijection $h\colon (0,1) \hookrightarrow\!\!\!\!\rightarrow \mathbb{R}$ by $h(x) :=$ ....

PA: Fri. 02 Dec $\mathbf{A} := \{(6,2), (2,7), (5,2), (7,3), (7,5), (3,4), (1,3), (1,5)\}$ is a binrel on $[1..7]$, with transitive closure $\mathbf{R}$. Then:

$2\mathbf{R}2$ is T F. $4\mathbf{R}6$ is T F. $7\mathbf{R}7$ is T F.

PB: Mon. 05 Dec *EoS 2022 Games Party*, from 11:45am–4pm, will take place at *Pascal's Cafe* on Wedn. 7Dec.

Circle **Yes!** **True!** **I-have-a-game!**

Games Party: !! Wed. 07 Dec Bring games and look photogenic, for our traditional Games Party, from 11:45 am to 4 pm, at Pascal's Cafe.