SeLo
MHF3202 8768 **Quizzes P** Monday 20Aug2018

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** <u>and</u> $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$. And $[-\infty \,..\, -1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathbf{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', <u>*but*</u> pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3+t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

SeLo [2018t] quizzes so far...

P1: Wed. 29 Aug $[\sqrt{3}^{\sqrt{8}}]^{\sqrt{2}}$ = ________. $\log_{64}(16)$ = ________.

P2: Fri. 31 Aug The four solutions to $[y-2]\cdot y \cdot [y+2] = -1/y$ are $y =$

____

[*Hint:* Apply the Quadratic Formula to y^2.]

P3: Wed. 05 Sep Write the truth-table for $B \Rightarrow [[\neg B] \Rightarrow C]$.

B	C	$\neg B$	$[\neg B] \Rightarrow C$	$B \Rightarrow [[\neg B] \Rightarrow C]$
F	*F*			
F	*T*			
T	*F*			
T	*T*			

P4: Fri. 14 Sep Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\ f(n) \subset \underbrace{\bigcup_{\ell=p-4}^{p+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E3$: ______ . $E2$: ______ . $E1$: ______ .

P5: Mon. 17 Sep

May lightning ↯ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	100	—	**1**	**0**
1	23		**0**	**1**
2				
3				
4				
5				

So GCD(100, 23) = ______ = [______ · 100] + [______ · 23].

P6: Mon. 08 Oct LBolt gives $G := \text{GCD}(23, 413) =$ ______ . And $23S + 413T = G$, where $S =$ ______ & $T =$ ______ are integers.

P7: Fri. 26 Oct Both $\sim$ and $\bowtie$ are equiv-relations on a set $\mathbb{Z}$. Define binrels **I** and **U** on $\mathbb{Z}$ as follows.

Define $\omega\, \mathbf{U}\, \lambda$ IFF *Either* $\omega \sim \lambda$ *or* $\omega \bowtie \lambda$ [or both].

Define $\omega\, \mathbf{I}\, \lambda$ IFF *Both* $\omega \sim \lambda$ *and* $\omega \bowtie \lambda$.

So "**U** *is an equiv-relation*" is: *T* *F*

So "**I** *is an equiv-relation*" is: *T* *F*

$\mathbf{A} := \{(6,2), (2,7), (5,2), (7,3), (7,5), (3,4), (1,3), (1,5)\}$ is a binrel on $[1..7]$, with transitive closure **R**. Then:

$2\mathbf{R}2$ is *T* *F*. $4\mathbf{R}6$ is *T* *F*. $7\mathbf{R}7$ is *T* *F*.

P8: Mod. 19 Nov An explicit bijection $\psi\colon \mathbb{Z} \hookrightarrow\!\!\!\rightarrow \mathbb{N}$ is this:

If $n \geq 0$, then $\psi(n) :=$ ______ .

If $n < 0$, then $\psi(n) :=$ ______ .

P9: Mod. 26 Nov To the interval $J := (-\frac{\pi}{2}, \frac{\pi}{2})$, define a bijection $g\colon (0,1) \hookrightarrow\!\!\!\rightarrow J$ by $g(x) :=$ ________________________________ .

Using this g and a trigonometric fnc, define a bijection $h\colon (0,1) \hookrightarrow\!\!\!\rightarrow \mathbb{R}$ by $h(x) :=$ ________________________________

Last SeLo quiz of semester!