

**Number Sets.** Expression  $k \in \mathbb{N}$  [read as “ $k$  is an element of  $\mathbb{N}$ ” or “ $k$  in  $\mathbb{N}$ ”] means that  $k$  is a natural number; a **natnum**. Expression  $\mathbb{N} \ni k$  [read as “ $\mathbb{N}$  *owns*  $k$ ”] is a synonym for  $k \in \mathbb{N}$ .

$\mathbb{N}$  = natural numbers =  $\{0, 1, 2, \dots\}$ .

$\mathbb{Z}$  = integers =  $\{\dots, -2, -1, 0, 1, \dots\}$ . For the set  $\{1, 2, 3, \dots\}$  of positive integers, the **posints**, use  $\mathbb{Z}_+$ . Use  $\mathbb{Z}_-$  for the negative integers, the **negints**.

$\mathbb{Q}$  = rational numbers =  $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$ . Use  $\mathbb{Q}_+$  for the positive rationals and  $\mathbb{Q}_-$  for the negative rationals.

$\mathbb{R}$  = reals. The **posreals**  $\mathbb{R}_+$  and the **negreals**  $\mathbb{R}_-$ .

$\mathbb{C}$  = complex numbers, also called the **complexes**.

For  $\omega \in \mathbb{C}$ , let “ $\omega > 5$ ” mean “ $\omega$  is **real** and  $\omega > 5$ ”. [Use the same convention for  $\geq, <, \leq$ , and also if 5 is replaced by any real number.]

Use  $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$ , the **extended reals**.

An “**interval of integers**”  $[b..c]$  means the intersection  $[b, c] \cap \mathbb{Z}$ ; ditto for open and closed intervals. So  $[e..2\pi] = \{3, 4, 5, 6\} = [3..6] = (2..6]$ . We allow  $b$  and  $c$  to be  $\pm\infty$ ; so  $(-\infty..-1]$  is  $\mathbb{Z}_-$ . And  $[-\infty..-1]$ , is  $\{-\infty\} \cup \mathbb{Z}_-$ .

Floor function:  $\lfloor \pi \rfloor = 3$ ,  $\lfloor -\pi \rfloor = -4$ .  
Ceiling fnc:  $\lceil \pi \rceil = 4$ . Absolute value:  $|-6| = 6 = |6|$  and  $|-5 + 2i| = \sqrt{29}$ .

**Mathematical objects.** Seq: ‘sequence’. poly(s): ‘polynomial(s)’. irred: ‘irreducible’. Coeff: ‘coefficient’ and var(s): ‘variable(s)’ and parm(s): ‘parameter(s)’. Expr.: ‘expression’. Fnc: ‘function’ (so ratfnc: means rational function, a ratio of polynomials). trnfn: ‘transformation’. cty: ‘continuity’. cts: ‘continuous’. diff’able: ‘differentiable’. CoV: ‘Change-of-Variable’. Col: ‘Constant of Integration’. Lol: ‘Limit(s) of Integration’. RoC: ‘Radius of Convergence’.

Soln: ‘Solution’. Thm: ‘Theorem’. Prop’n: ‘Proposition’. CEX: ‘Counterexample’. eqn: ‘equation’. RhS: ‘RightHand side’ of an eqn or inequality. LhS: ‘lefthand side’. Sqrt or Sqroot: ‘square-root’, e.g., “the sqroot of 16 is 4”. Ptn: ‘partition’, but pt: ‘point’ as in “a fixed-pt of a map”.

Binop: ‘Binary operator’. Binrel: ‘Binary relation’.

FTC: ‘Fund. Thm of Calculus’. IVT: ‘intermediate-Value Thm’. MVT: ‘Mean-Value Thm’.

The **logarithm** function, defined for  $x > 0$ , is  $\log(x) := \int_1^x \frac{dv}{v}$ . Its inverse-fnc is  $\exp()$ .

For  $x > 0$ , then,  $\exp(\log(x)) = x = e^{\log(x)}$ . For real  $t$ , naturally,  $\log(\exp(t)) = t = \log(e^t)$ .

**PolyExp:** ‘Polynomial-times-exponential’, e.g.,  $[3 + t^2] \cdot e^{4t}$ . PolyExp-sum: ‘Sum of polyexps’. E.g.,  $f(t) := 3te^{2t} + [t^2] \cdot e^t$  is a polyexp-sum.

**Phrases.** WLOG: ‘Without loss of generality’. IFF: ‘if and only if’. TFAE: ‘The following are equivalent’. ITOF: ‘In Terms Of’. OTForm: ‘of the form’. FTSOC: ‘For the sake of contradiction’. And  $\otimes$  = “Contradiction”.

IST: ‘It Suffices To’, as in ISTShow, ISTExhibit.

Use w.r.t: ‘with respect to’ and s.t: ‘such that’.

**Latin:** e.g. *exempli gratia*, ‘for example’. i.e. *id est*, ‘that is’. N.B: *Nota bene*, ‘Note well’. inter alia: ‘among other things’. QED: *quod erat demonstrandum*, meaning “end of proof”.

## SeLo [2017g] quizzes so far...

**Q1:** Wed. 11 Jan The slope of line  $3[y - 5] = 2[x - 2]$  is .....  
Point  $(-4, y)$  lies on this line, where  $y =$  .....

$$[\sqrt{6}^{\sqrt{8}}]^{\sqrt{2}} = \dots \quad \log_{125}(5) = \dots$$

**Q2:** Fri. 13 Jan

May lightning ⚡ strike this table! (I.e, please fill in.)

$n$	$r_n$	$q_n$	$s_n$	$t_n$
0	100	—	1	0
1	23		0	1
2				
3				
4				
5				

So GCD(100, 23) = ..... = [ ..... · 100] + [ ..... · 23].

**Q3:** Wed.  
18 Jan

May lightning ⚡ strike this table! (I.e, please fill in.)

$n$	$r_n$	$q_n$	$s_n$	$t_n$
0	109	—	1	0
1	21		0	1
2				
3				
4				

So  $\text{GCD}(109, 21) = \underline{\hspace{2cm}} = [\underline{\hspace{2cm}} \cdot 109] + [\underline{\hspace{2cm}} \cdot 21]$ .

And  $x = \underline{\hspace{2cm}} \in (-50 .. 50]$  solves congr.  $21x \equiv_{109} 3$ .

**Q4:** Fri.  
20 Jan LBolt:  $\text{GCD}(70, 30) = \underline{\hspace{2cm}} \cdot 70 + \underline{\hspace{2cm}} \cdot 30$ .

So (LBolt again)  $G := \text{GCD}(70, 30, 42) = \underline{\hspace{2cm}}$  and  
 $\underline{\hspace{2cm}} \cdot 70 + \underline{\hspace{2cm}} \cdot 30 + \underline{\hspace{2cm}} \cdot 42 = G$ .

**Q5:** Fri.  
03 Feb Coeff of  $x^7 y^{15}$  in  $[2x + y^3 + 5]^{20}$  is  $\underline{\hspace{2cm}}$ .

[You may leave your answer as a product of *posints*, or you may multiply-out.]

**Q6:** Wed.  
22 Feb Below,  $G$  and  $\Omega$  are sets. Then

$$G^\Omega = \left\{ \underline{\hspace{2cm}} \mid \underline{\hspace{2cm}} \right\}.$$

Suppose  $R$  is a binrel from  $G$  to  $\Omega$ . Then

$$\text{CoRange}(R) = \left\{ \underline{\hspace{2cm}} \mid \underline{\hspace{2cm}} \right\}.$$

**Q7:** Wed.  
22 Feb With  $f(x) := x^2$  and  $g(x) := x + 3$ ,

then  $[f \triangleright g](5) = \underline{\hspace{2cm}}$ .

Circle those operators/relations which are chiral:

$\neq$   $\bullet$   $\circ$   $\text{Max}$   $\div$   $\leq$   $<$   $\wedge$

**Q8:** Fri.  
24 Feb In Gale's game of CHOMP [whose  $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$  version is still unsolved], eating the Poison-Pill Circle: Wins Loses

CHOMP, with best-play on a *finite* rectangular board, is a first-player win:  $\overline{T} \quad F$

**Q9:** Fri.  
03 Mar To the interval  $J := (-\frac{\pi}{2}, \frac{\pi}{2})$ , define a bijection  $g: (0, 1) \hookrightarrow J$  by  $g(x) := \underline{\hspace{2cm}}$ .

Using this  $g$  and a trigonometric fnc, define a bijection  $h: (0, 1) \hookrightarrow \mathbb{R}$  by  $h(x) := \underline{\hspace{2cm}}$ .

**QA:** Mon.  
13 Mar A “Cantor’s-Hotel” type bijection  $f: (5, 6] \hookrightarrow (0, 1)$  is:

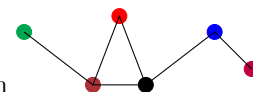
$f(\underline{\hspace{2cm}}) := \underline{\hspace{2cm}}$ , for each posint  $n$ ;

and  $f(x) := \underline{\hspace{2cm}}$ , for each  $x \in (5, 6] \setminus C$ ,

where  $C := \underline{\hspace{2cm}}$ .

**QB:** Wed.  
12 Apr

*Bird with a broken wing:* Graph



has Chromatic

poly  $\mathcal{P}(x) = \underline{\hspace{2cm}}$ .

[Can be done by inspection. Do not bother to multiply out.]

**QC:** Mon.  
17 Apr

*Trapezoid:*

Graph has

Chromatic

poly  $\mathcal{P}(x) = \underline{\hspace{2cm}}$ .

[Edge-glue three copies of  $K_3$ . Do not multiply out, but do write as a product of polynomials..]

*Games Party!*