Q1: (Fri, 4Sep) Let $\boldsymbol{\tau}()$ and $\boldsymbol{\sigma}()$ be the number-of and sum-of divisors, resp.. Then $\boldsymbol{\tau}(2700) =$ ________

and $\boldsymbol{\sigma}(2700) =$ ________. (Please leave each answer as a product of three integers.)

Q2: (Fri, 11Sep) a Write $27^{2009} \equiv_7$ __ (i.e, working mod 7) and $9^{35} \equiv_7$ ________, each as a value in $[0..7)$.

[*Hint:* This can be done by inspection.]

b If $5^K \bullet\| 1000$, then $K=$ ________.

Q3: (Wed, 16Sep) May lightning ↯ strike this table! (I.e, please fill in.)

n	r_n	q_n	s_n	t_n
0	100	—	**1**	**0**
1	23		**0**	**1**
2				
3				
4				
5				

So $\text{GCD}(100, 23) =$ ____ $= [$ ____ $\cdot 100] + [$ ____ $\cdot 23]$.

And $x=$ ________ $\in [0..100)$ solves congr. $23x \equiv_{100} 8$.

Q4: (Wed, 23Sep) a Write the name of each symbol: $\lfloor$ *floor* $\lceil$ *ceiling*

$\langle$ ________ $\{$ ________ $[$ ________.

b Write the free vars in each of these expressions.

$$\exists n \in \mathbb{N}\colon\ f(n) \subset \overbrace{\underbrace{\bigcup_{\ell=r-4}^{r+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E1}}_{E2}}^{E3}$$

$E1$: ________. $E2$: ________. $E3$: ________.

Q5: (Fri, 25Sep) i Stmt $C \Rightarrow B$ has *contrapositive* ________ and *converse* ________. Recall &, $\vee$, $\neg$ mean AND, OR, NOT. Using *only* symbols $\wedge, \vee, \neg, \boldsymbol{B}, \boldsymbol{C},], [$, write $C \Rightarrow B$ as ________.

ii Let $B:=39, M:=105, T:=3$. Cgr $B \cdot x \equiv_M T$ has *reduced Cgr* $\beta \cdot y \equiv_\mu \tau$, where $\beta=$ ________, $\mu=$ ____, $\tau=$ ____, and soln $y=$ ____ $\in [0..\mu)$.

Cgr $B \cdot x \equiv_M T$, has ____ many solns, which are $x=$ ________ $\in [0..M)$.

Q6: (Mon, 05Oct) a Euler $\varphi(121000) =$ ________. Express your answer as a product $p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

b [With $\mathcal{P}()$ the *powerset* operator, let $S :=$ 3-stooges.] Then $|\mathcal{P}(S)| =$ ____ and $|\mathcal{P}(\mathcal{P}(S))| =$ ________.

Q7: (Fri, 9Oct) The mod-13 reciprocal of 5 is ________.

Q8: (Wed, 14Oct) a Repeating decimal $3.1\overline{41}$ equals $\frac{n}{d}$, where posints $n \perp d$ are $n=$ ________ and $d=$ ________.

b Writing 167/111 as a repeating decimal gives $\frac{167}{111} =$ ________.

Q9: (Fri, 23Oct) The number of ways of having 9 objects from 4 types is $\left[\!\left[\begin{matrix}4\\9\end{matrix}\right]\!\right] \overset{\text{Binom}}{\underset{\text{coeff}}{=}} \binom{\quad}{\quad} \overset{\text{Integer}}{\underset{\text{numeral}}{=}}$ ________.

And $\left[\!\left[\begin{smallmatrix}4\\9\end{smallmatrix}\right]\!\right] = \left[\!\left[\begin{smallmatrix}T\\N\end{smallmatrix}\right]\!\right]$, where $T=$ ________ $\neq 4$, and $N=$ ________.

Q10: (Fri, 30Oct) ?

Whoa! See next page for potential quiz/exam probs.

Potential quiz problems

I have deleted problems that either occur in link "SeLo-B" on

http://www.math.ufl.edu/~squash/course.selo.2009t.html

or which have already occured on quizzes and are up above.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

P1: i Prove, for each $n \in [3 .. \infty)$, that $2n + 1 \leqslant n^2$. [*Hint:* Either use induction on n, or just complete the square.]

ii Consider $\boxed{*\colon \forall n \in [K .. \infty)\colon n^2 < 2^n.}$ Then K:= ___ is the smallest natnum st. $(*)$. Prove $(*)$ by induction on n.

P2: Let $L(n) := n[n+1]$. And let $R(n) := 2 \cdot \sum_{k=1}^{n} k$. By induction on n, prove that $\boxed{\forall n \in \mathbb{N}\colon\ L(n) = R(n)}$. *Explicitly* prove the base case. *Explicitly* state the induction implication, then prove that it holds for each $n \in \mathbb{N}$.

P3: Let $L(n) := 2n^3 + 3n^2 + n$. And let $R(n) := 6 \cdot \sum_{k=1}^{n} k^2$. By induction on n, prove that $\boxed{\forall n \in \mathbb{N}\colon\ L(n) = R(n)}$. *Explicitly* prove the base case. *Explicitly* state the induction implication, then prove that it holds for each $n \in \mathbb{N}$.

P4: Let $L(k) := [5^{[2k]}] - 1$. By induction on k, prove that $\boxed{\forall k \in \mathbb{N}\colon\ L(k) \bullet\!| \ 3}$.

P5: By strong induction, prove: THM. *For each posint n, there exists a finite multiset S_n of prime numbers such that $\prod(S_n) = n$.*

Baby Binomial-Thm. *For each posint N, we have the fol. equality between polynomials in x and y:*

$[x+y]^N$= ___ .

[*Hint:* Don't use a "..." to hide the general term in your expression. Write a precise expr., using a big-op.]

By Baby Binomial-Thm, $9^4 + [4 \cdot 9^3] + [6 \cdot 9^2] + [4 \cdot 9] + 1$ equals B^C, where B= ___ and C= ___ .

And $9^4 - [4 \cdot 9^3] + [6 \cdot 9^2] - [4 \cdot 9] + 1 = Q^R$, where Q= ___ and R= ___ .

On $\mathbb{Z}_+$, write $x \$ y$ IFF $xy < 0$. So \$ is *Circle*

Transitive: T F. **Symm.**: T F. **Reflex.**: T F.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x - y \leqslant 1$. Then ∇ is:

Trans.: T F. **Symm.**: T F. **Reflex.**: T F.

(Be *careful* on both parts!)

Consider natnums $J_1, \ldots, J_K$ and $N := \sum_{\ell=1}^{K} J_\ell$. The ***multinomial coefficient*** $\binom{N}{J_1,\ldots,J_K}$ *means*

___ .

Using factorials, $\binom{N}{\vec{\mathbf{J}}}$= ___ .

$\binom{10}{2,3,5}$ $\underline{\underline{\text{Single}}}$ integer ___ . $\binom{13}{1,2,4,4}$= ___ .

Hint: The following can be solved by LBolting twice.

LBolt: GCD(70, 42)= ___ $\cdot 70$ + ___ $\cdot 42$.

So (LBolt again) $G :=$ GCD(70, 42, 60)= ___ and ___ $\cdot 70$ + ___ $\cdot 42$ + ___ $\cdot 60 = G$.

$\sum_{n=0}^{67} 4^n \cdot \binom{67}{n} = B^C$, where B= ___ and C= ___ .

$\sum_{\ell=0}^{25} \binom{51}{\ell} = D^E$, where D= ___ and E= ___ .

[*Hint:* Baby-Binomial for both. Note .]

Let B_0:=385, B_1:=105, B_2:=165 and B_3:=231. Thus $G :=$ GCD(B_0,B_1,B_2,B_3)= ___ . [*Hint:* LBolt thrice.] Thus $[\sum_{j=0}^{3} s_j B_j] = G$, where

s_0= ___ , s_1= ___ , s_2= ___ , s_3= ___ .

On $\mathbb{R}_+$, define several relations: Say that $x\mathfrak{R}y$ IFF $y - x < 17$. Define $\mathfrak{P}$ by: $x\mathfrak{P}y$ IFF $x^{\log(y)} = 5$. Say that $x\mathfrak{I}y$ IFF $x + y$ is irrational.

Use $\bullet\!|$ for the "divides" relation on the positive integers: $k \bullet\!| \ n$ iff there exists a posint r with $rk = n$.

'1 Please circle those of the following relations which are *transitive* (on their domain of defn).

$\neq$ $\bullet\!|$ $\leqslant$ $\mathcal{R}$ $\mathcal{P}$ $\mathcal{I}$

‘2 Circle the *symmetric* relations:

$\neq$ $\bullet\!|$ $\leqslant$ $\mathcal{R}$ $\mathcal{P}$ $\mathcal{I}$

‘3 Circle the *reflexive* relations:

$\neq$ $\bullet\!|$ $\leqslant$ $\mathcal{R}$ $\mathcal{P}$ $\mathcal{I}$

Let $B{:=}115, M{:=}245, T{:=}10$. Congruence $B{\cdot}x \equiv_M T$ has *reduced congruence* $\beta{\cdot}y \equiv_\mu \tau$, where $\beta=$ ________, $\mu=$ ________, $\tau=$ ________, and soln $y=$ ________ $\in[0\,..\,\mu)$.

Congruence $B{\cdot}x \equiv_M T$, has ____ many solns, which are $x=$ ________________________________ $\in[0\,..\,M)$.

Repeating decimal $1.23\overline{456}$ equals $\frac{n}{d}$, where posints $n \perp d$ are $n=$ ________________ and $d=$ ________________.

Writing 7/531 as a repeating decimal gives $\frac{7}{531} =$ ________________________________.

A posint N is ***perfect*** IFF [Defn] ________________

____

Thm: *An* _even_ *posint* N *is perfect IFF* (state the thm)

____

____

(This thm is credited to ____________ and ____________.)

Applying the theorem, the 4^{th} even perfect number equals $A{\cdot}B$ where $A=$ ____________ and $B=$ ____________.

Using set-builder notation, define the set of primes.

Primes = $\{$n ∈ WHAT | Conditions on n$\}$, using some of the symbols

such that, if, then, and, or, not, $0\;1\;2\ldots$

$\forall\;\exists\;\nexists\;\in\;\mathbb{N}\;\mathbb{Z}_+\;[a\,..\,b)\;\bullet\;+\;=$

and *avoiding* “*factor(s), divides, is-a-multiple, splits, irreducible, composite, Gcd, Lcm* ...” and similar, uh, cheats. Every quantification must specify its set!

For $k,n \in \mathbb{Z}_+$, let $L := \text{LCM}(k,n)$ & $G := \text{GCD}(k,n)$. So $L=$ __ (A Formula(G,k,n) using mult & div.)

Thus $\text{LCM}(925, 777)=$ ________________ (write a *product of integers*). [*Hint:* Use LBolt to compute $\text{GCD}(925, 777)$, etc.]

Write $108^3 \equiv_5$ ________________ (i.e, working mod 5) and $46^{2787} \equiv_5$ ____________, each as a value in $[0\,..\,5)$. [*Hint:* This can be done by inspection.]

Expression “$K \perp N$” *means* ________________

____.

An example of such a pair, with $14 \leqslant K \leqslant 16$ and $32 \leqslant N \leqslant 36$, is $K=$ __________ & $N=$ __________.

The author of our text is ____________________.

Material not yet reached. Some of these problems refer to material we have not yet covered, and use terminology that we have not yet learned. The order that problems appear below does not necessarily correspond to the order in which we will study material.

Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\boldsymbol{i}}=$ __________ $+\boldsymbol{i}\cdot\Big[$ __________ $\Big]$.

Thus $\dfrac{5-\boldsymbol{i}}{2+3\boldsymbol{i}}=$ ______________ $+\boldsymbol{i}\cdot\Big[$ ______________ $\Big]$.

By the way, $|5-3\boldsymbol{i}|=$ ______________.

Note $[1+\boldsymbol{i}]^{86}=\Big[$ __________ $\Big]+\boldsymbol{i}\cdot\Big[$ __________ $\Big]$.

[*Hint: Multiplying complexes multiplies their moduli, and adds their angles.*]

Sqroot of $\boldsymbol{i}-1$ in the upper-halfplane is $r\cdot\text{cis}(\theta)$, where $r=$ __________ $\in\mathbb{R}_+$ and $\theta=$ __________ $\in[0,\frac{\pi}{2}]$.